TOPOLOGICAL METHODS IN ALGEBRAIC GEOMETRY

Subtitle: Can the angel of topology live happily together with the devil of abstract algebra?*

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Abstract:
Algebraic topology flourished from some of its applications (such as Brouwer's fixed point theorem, or the theorem of Borsuk-Ulam) showing the non-existence of certain continuous maps since they would imply the existence of homomorphisms satisfying algebraic properties which are manifestly impossible.
Conversely, the theory of fibre bundles and homotopy theory give a topological incarnation of a group G through its classifying space BG. The theory of classifying spaces translates then group homomorphisms into continuous maps to classifying spaces.
In algebraic geometry, the theory of Albanese varieties can be understood as dealing with the case where G is free abelian.
For more general G, an important question is the one of the regularity of these classifying maps, and their complex analyticity. These questions, which were addressed in the last 40 years, have powerful applications to moduli theory.
After this general introduction, I shall concentrate on some classes of projective varieties which are classifying spaces for some group, and on their rigidity and quasi-rigidity properties. I shall show how the investigations of their moduli spaces lead to some group theoretical questions, and to the investigation of moduli spaces of curves with symmetries.
Key examples are: varieties isogenous to a product, and the Inoue-type varieties introduced in recent work with Ingrid Bauer: for these the moduli space is determined by the topological type.
I shall devote the final part to:
a) recent joint work with Michael Lönne and Fabio Perroni, dealing with the irreducible components of moduli spaces of curves with symmetries, with stabilization results and open questions,
b) an application of these ideas to provide faithful actions of the absolute Galois group $Gal(\bar{Q} / Q)$ on homeomorphism types of algebraic varieties (joint work with Ingrid Bauer, Fritz Grunewald**)
* Historical note:
In these days the angel of topology and the devil of abstract algebra fight for the soul of each individual mathematical domain. [Hermann Weyl (1939), p.500 of "Invariants", Duke Mathematical Journal 5 (3): 489-502.]