ABSTRACT

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Singular Poisson-Kähler geometry of stratified Kähler spaces and quantization

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In the presence of classical phase space singularities the standard methods are insufficient to attack the problem of quantization. In certain situations these difficulties can be overcome by means of the notion of *stratified Kähler space*.

Examples include certain moduli spaces of holomorphic vector bundles on a Riemann surface as well as the closure of a holomorphic nilpotent orbit, a special case being reduced spaces arising from angular momentum. Projectivization of holomorphic nilpotent orbits yields exotic stratified Kähler structures on complex projective spaces and on certain complex projective varieties including complex projective quadrics. Symplectic reduction, applied to Kähler manifolds, yields a particular class of examples; this includes adjoint and generalized adjoint quotients of complex semisimple Lie groups which, in turn, underly certain lattice gauge theories.

A suitable holomorphic quantization procedure on stratified Kähler spaces unveils a certain *quantum structure having the classical singularities as its shadow*. The new structure which thus emerges is that of a *costratified Hilbert space*, that is, a Hilbert space together with a system which consists of the subspaces associated with the strata of the reduced phase space and of the corresponding orthoprojectors. The costratified Hilbert space structure reflects the stratification of the reduced phase space.

We will illustrate the approach with a quantum (lattice) gauge theory which involves certain classical singularities.

References

[1] J. Huebschmann: Kähler spaces, nilpotent orbits, and singular reduction. Memoirs of the AMS **172 (814)**, Amer. Math. Soc., Providence R.I., , 2004, `math.DG/0104213`

[2] J. Huebschmann: Kähler quantization and reduction. J. reine angew. Math. **591** (2006), 75–109, `math.SG/0207166`

[3] J. Huebschmann: Singular Poisson-Kähler geometry of stratified Kähler spaces and quantization. Travaux mathématiques **19** (2011), 27–63, `arxiv:1103.1584`

[4] J. Huebschmann, G. Rudolph and M. Schmidt: A gauge model for quantum mechanics on a stratified space. Commun. Math. Physics **286** (2009), 459–494, `hep-th/0702017`