K3 surfaces and modular forms

K3 surfaces continue to be of great importance for diverse fields of mathematics and physics such as algebraic geometry, differential geometry, number theory, and string theory. In this talk, I will concentrate on the connection between K3 surfaces and modularity which features many distinguished properties of K3 surfaces.

Modularity refers to classical modular forms on the upper half plane. A classical construction of Shimura associates an elliptic curve over **Q** to every Hecke eigenform of weight 2 with rational coefficients. The converse statement that every elliptic curve over **Q** is modular is the Taniyama-Shimura-Weil conjecture, proven by Wiles, Taylor et al.

For higher weight, however, the opposite situation applies: Nowadays we know modularity for some classes of varieties, but it is an open problem whether all newforms of fixed weight with rational coefficients can be realised in a single class of varieties. I will review the historical developments and discuss joint work with Noam Elkies that uses K3 surfaces for the first solution to the realisation problem in higher weight.

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