Abstract: Operads, interchange, iterated loop space structures, and the dual Deligne Conjecture.

Operads encode algebraic structures. Originally they were introduced to study iterated loop spaces. The algebraic structure of an n-fold loop space $\Omega^n X$ is encoded by a so-called E_n-operad. Each n-fold loop space has an E_n-structure and each connected space with an E_n-structure is of the weak homotopy type of an n-fold loop space.

Now $\Omega^n X$ has n interchanging single loop space structures. So a space having an E_n-structure should have n interchanging E_1-structures and a space having an E_k- and an E_l-structure which interchange morally should be an E_{k+l}-space. Unfortunately, the situation is not that easy. We state two conjectures, but we are still far away from proving them. The analysis of a special case allows us to verify the dual of the Deligne Conjecture.

There are algebraic versions of E_n-structures, and in 1993 Deligne asked whether the Hochschild cochain complex of an associative ring has a natural E_2-structure. This conjecture has been proved and extended to other categories such as categories of highly structured spectra in homotopy theory by a number of people. We prove the dual statement by showing that the topological Hochschild homology spectrum $THH(R)$ of an E_{n+1}-ring spectrum R is an E_n-ring spectrum.

In the talk we explain the notions of the title, study some interchange results in detail, and sketch the proof of the dual Deligne Conjecture.

Remarks: (1) The result about $THH(R)$ has also been obtained by Maria Basterra and Mike Mandell using different methods.

(2) The talk is about joint work with Morten Brun and Zig Fiedorowicz.