Geometry of noncommutative 'spaces' and schemes.

Abstract:

Many 'spaces' which appear in noncommutative algebraic geometry are represented by categories of different type – abelian, exact, triangulated, A-infinity and DG-categories; and morphisms of 'spaces' are represented by functors thought as inverse image functors. Thus, at the starting point, we get instead of geometry its categorical reflections. These reflections turn to be sufficiently rich to define noncommutative versions of basic objects and notions of algebraic geometry, including noncommutative schemes, smooth morphisms etc., to obtain basic facts about these notions (including flat descent) and produce a number of interesting examples. I will give a short outline of the pseudo-geometric picture in the first part of the lecture. The main stress of its second part is that pseudo-geometric 'spaces' have canonical spectral theories and the choice of a spectral theory implies a *geometric realization* of 'spaces'. I will sketch, the geometry of 'spaces' represented by abelian and (if time permits) triangulated categories. This geometry gives tools for local study of noncommutative schemes and more general locally affine 'spaces' and has immediate applications (and gives new insights) in representation theory.

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