Classification and Statistics of Finite-Index Subgroups in Free Products (The General Poincaré-Klein Problem)

Abstract

We report on the solution of the problem mentioned in the title, whose formulation goes back to work of Poincaré and Klein on automorphic functions in the 1880s. Specifically, the realization, asymptotic, and distribution problems for isomorphism types of finite-index subgroups are discussed in a free product of the form

$$\Gamma = G_1 * G_2 * \cdots * G_s * F_r,$$

where $G_1, G_2, \ldots, G_s$ are finite groups and F_r is free of rank r. The results generalize previous work (*Adv. Math.* **188** (2004), 1–50), which dealt with finite-index subgroups in groups of the form

$$\Gamma = C_{p_1}^{*e_1} * C_{p_2}^{*e_2} * \cdots * C_{p_t}^{*e_t} * F_r,$$

where $p_1, p_2, \ldots, p_t$ are distinct primes. Apart from group-theoretic arguments, the proofs make use of asymptotic, number-theoretic, combinatorial, and probabilistic ideas and techniques.

Prof. Dr. T. W. Mueller (Queen Mary University of London)