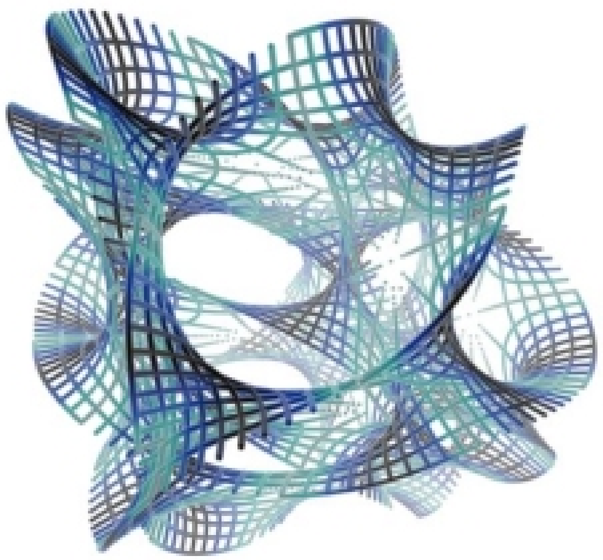
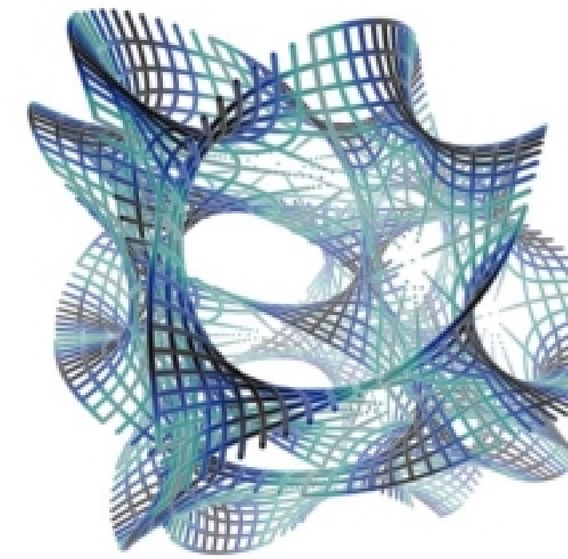
Geometry, Non-Geometry and String Theory



Special Geometric Structures in Mathematics & Physics

Hamburg, 2014

Plan of Lectures



- Overview
- Generalized Geometry and all that
- T-duality
- T-folds, non-geometric backgrounds. Doubled sigma-models
- Double Field Theory

Geometric Structures

- Riemannian: manifold with metric (M, g)
- Metric + p-form gauge field (M, g, C_p)
- 1-form field: Connection on Bundle over M
- p-form gauge field: Gerbe Connection
- Metric + 2-form: Generalised Geometry
- Metric + p-form: Extended Geometry

Structures in D dimensions

- Generalised Geometry: metric + B-field, natural action of O(D,D), tangent space "doubled" to $T \oplus T^*$ Hitchin
- Extended Geometry: metric+p-form+..., natural action of some group, typically an exceptional group, tangent space extended to $T \oplus \Lambda^{p-1}T^* \oplus \ldots$
- e.g. D=6, p=2: G= E_6, D=7, p=2: G= E_7

CMH
Waldram + ...

Gravity, Supergravity and Strings

- Gravitational field: metric on spacetime
- Supergravity: metric plus p-form gauge fields
- Sugra effective part of full quantum theory: superstrings or M-theory. String theory has infinite tower of massive fields
- Physics: clues and insights to new geometry
  Geometry: helps solve physics

Strings in Background

Manifold, background tensor fields G_{ij}, H_{ijk}, Φ

String propagation determined by sigma-model

$$S = \int d^2\sigma \sqrt{h} \left(G_{ij} h^{ab} \partial_a X^i \partial_b X^j + B_{ij} \epsilon^{ab} \partial_a X^i \partial_b X^j + \Phi R \right)$$

Target space coordinates X^i

World-sheet coordinates $\sigma^a = (\sigma, \tau)$

Fields $X^i(\sigma^a)$ determine embedding

H=dB

Conformal invariance of 2-d theory constrains

G_{ij}, H_{ijk}, Φ - Gives target space field equations.

Quantum sigma model then defines a conformal field theory (CFT)

Symmetries

- (G, B, Φ) satisfying field eqns determine CFT
- Same CFT can be given by $(G, B, \Phi), \;\; (G', B', \Phi')$
- - if related by diffeos + B-field gauge transformations
- - if related by T-duality
- Same physics, so these are SYMMETRIES

T-duality

- Symmetry of string theory on S^1 or torus, not a symmetry of field theory
- Takes S^1 of radius R to S^1 of radius 1/R
- Exchanges momentum p and winding w
- Exchanges S^1 coordinate X and dual S^1 coordinate $\tilde{X}$
- Acts on "doubled circle" with coordinates $(X, \tilde{X})$

Symmetry & Geometry

- Spacetime constructed from local patches
- Symmetries of physics used in patching
- Reparameterisation symmetry: patching with diffeomorphisms, to give manifold
- Patching with gauge symmetries: bundles
- String theory has new symmetries, not present in field theory. New geometries.
- Non-geometric string backgrounds

- T-duality: perturbative symmetry on torus
- U-duality: non-perturbative symmetry of type II strings or M-theory on torus, exceptional groups
- Mirror Symmetry: perturbative symmetry of superstring on Calabi Yau
- Use in patching: T-folds, U-folds, Mirror-folds CMH

String Theory in Minkowski Space

Infinite set of fields

$$g_{ij}(x), b_{ij}(x), \phi(x), \ldots, C_{ijk\ldots l}(x), \ldots$$

Finite set of massless fields +
infinite tower of massive fields

Integrating out massive fields gives field theory for

$$g_{ij}(x), b_{ij}(x), \phi(x)$$

Strings on a Torus

- States: momentum p, winding w
- String: Infinite set of fields $\psi(p, w)$
- Fourier transform to doubled space: $\psi(x, \tilde{x})$
- "Double Field Theory" from closed string field theory. Some non-locality in doubled space
- Infinite set of fields in doubled space

String Theory on a Torus

Infinite set of double fields

$$g_{ij}(x,\tilde{x}), b_{ij}(x,\tilde{x}), \phi(x,\tilde{x}), \ldots, C_{ijk\ldots l}(x,\tilde{x}), \ldots$$

- Fields on double space satisfy differential constraint
- T-duality symmetry
- Coordinates doubled, but tensor indices not

Seek **double field theory** for

$$g_{ij}(x,\tilde{x}), b_{ij}(x,\tilde{x}), \phi(x,\tilde{x})$$

- Novel symmetry, reduces to diffeos + B-field trans. in *any* half-dimensional subtorus
- Backgrounds depending on $\{x^a\}$ seen by particles, on $\{\tilde{x}_a\}$ seen by winding modes.
- Captures exotic and complicated structure of interacting string: Non-polynomial, algebraic structure homotopy Lie algebra, cocycles.
- T-duality symmetry manifest
- Formalism for T-folds etc
- Generalised T-duality: no isometries needed

Earlier work on double fields:
Siegel, Tseytlin

Double Field Theory

Hull & Zwiebach

- Double field theory on doubled torus
- General solution of string theory: involves doubled fields $\psi(x, \tilde{x})$
- DFT needed for non-geometric backgrounds
- *Real* dependence on *full* doubled geometry, dual dimensions not auxiliary or gauge artifact. Double geom. *physical* and *dynamical* (with weak constraint)
- For torus fibrations, fibres doubled. What happens for more general spaces?

- Generalized Geometry: double tangent space to manifold M

- Double Geometry: Double manifold M (if M is torus). Tangent space doubles too. (Double fibres for torus fibration)

- DFT: Fields in doubled space. (With strong constraint, truncates locally to field theory written in terms of generalised geometry.)

- Non-geometric backgrounds: string theory solns., not manifolds with smooth fields. Double geometry provides a formulation.

Generalised Geometry

Studies structures on a d-dimensional manifold M on which there is a natural action of O(d,d) Hitchin

$T \oplus T^*$ Vector + 1-form

$$V = v + \xi \in T \oplus T^*$$

$$V^I = \begin{pmatrix} v^i \\ \xi_i \end{pmatrix}$$

O(d,d) Metric

$$\eta(v + \xi, v + \xi) = 2\iota_v \xi$$

$$\eta = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

O(d,d) $V \to gV$ $g^t \eta g = \eta$

$$\iota_v \xi = v^i \xi_i$$

Generalised Metric $\mathcal{H}_{IJ}$

Gualtieri

Positive definite metric on $T \oplus T^*$ compatible with η

$$\eta^{-1}\mathcal{H}\eta^{-1} = \mathcal{H}^{-1}$$

$S = \eta^{-1}\mathcal{H}$ satisfies $S^2 = \mathbb{1}$ Real structure

Parameterised by $G = G^t, \ B = -B^t$ $\qquad E = G + B$

Metric G and B-field B

$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$$

O(d,d):

$\mathcal{H} \to g^t \mathcal{H} g \qquad E \to (aE + b)(cE + d)^{-1}$

Parameterise coset $\qquad \dfrac{O(d,d)}{O(d) \times O(d)}$

Geometric Subgroup

$$g = \begin{pmatrix} M & 0 \\ \Theta & (M^t)^{-1} \end{pmatrix}$$

$$G \to M^t G M, \qquad B \to M^t B M \qquad GL(d, \mathbb{R})$$

$$B \to B + \Theta$$ B-shifts $$\Gamma(\mathbb{R}) \subset O(d,d)$$

Transition Functions

$$T, \; T^*, \; T \oplus T^* \ldots.$$

$$GL(d, \mathbb{R}) \qquad \frac{\partial x'^i}{\partial x^j}$$

Symmetries

Generalised Tangent Bundle:

Transition functions $\Gamma(\mathbb{R})$ Θ exact

$$B' = B + d\lambda$$

Rest of O(d,d) not symms, can't be used as transitions

Suggestive to think of $T \oplus T^*$ as O(d,d) bundle

Really $GL(d, \mathbb{R})$ bundle

Transition Functions $GL(d, \mathbb{R})$ $\frac{\partial x'^i}{\partial x^j}$

Generalised Tangent Bundle:

Transition functions $\Gamma(\mathbb{R})$

Rest of O(d,d) not symms, can't be used as transitions

If M has an n-torus fibration, string theory has $O(n, n; \mathbb{Z})$ T-duality symmetry. Suggests using this for transition functions in string theory

T-fold

CMH

Type I Extended Geometry

Action of O(d,d)

$T \oplus T^*$

cf Brane charges

Generalised metric

$G, B_2 \quad \Longleftrightarrow \quad \mathcal{H}_{IJ} \in \frac{O(d,d)}{O(d) \times O(d)}$

Type I Extended Geometry

Action of O(d,d)

$T \oplus T^*$

cf Brane charges

Generalised metric

$G, B_2 \longleftrightarrow$

$$\mathcal{H}_{IJ} \in \frac{O(d,d)}{O(d) \times O(d)}$$

Type II: O(d,d) $\longrightarrow$ E_{d+1}, add RR fields

Type I Extended Geometry

Action of O(d,d)

$T \oplus T^*$

cf Brane charges

Generalised metric

G, B_2 $\Longleftrightarrow$ $\mathcal{H}_{IJ} \in \frac{O(d,d)}{O(d) \times O(d)}$

Type M: O(d,d) $\Rightarrow$ E_d, $B_2 \Rightarrow C_3$

$T \oplus T^* \Rightarrow T \oplus \Lambda^2 T^*$

Type M Extended Geometry $d \leq 4$

Action of E_d

$T \oplus \Lambda^2 T^*$

cf Brane charges

Generalised metric

$G, C_3 \quad \Longleftrightarrow \quad \mathcal{H}_{IJ} \in \dfrac{E_d}{H_d}$

Type M Extended Geometry $d \leq 7$

Action of E_d

$$T \oplus \Lambda^2 T^* \oplus \Lambda^5 T^* \oplus \Lambda^6 T$$

cf Brane charges

$$G, C_3, \tilde{C}_6 \quad \Longleftrightarrow$$

Generalised metric

$$\mathcal{H}_{IJ} \in \frac{E_d}{H_d}$$

Symmetries and T-Duality

Spacetime: Torus x N

$M = T^d \times N$ N some non-compact space

Gravity: Symmetry GL(d,Z)
from large diffeomorphisms on T
Modes with dependence on T have
quantized momentum (Fourier modes)

Effective theory on N: Symmetry GL(d,R);
fields with no dependence on T

$$M = T^d \times N$$

Gravity + B-field

Effective theory on N: Symmetry O(d,d)

Full quantum theory: **String theory**

Symmetry broken to O(d,d;Z), contains GL(d,Z)

Exact **T-duality** symmetry of string theory

Includes strange non-geometric transformations

$$M = T^d \times N$$

Gravity + B-field

Effective theory on N: Symmetry O(d,d)

Full quantum theory: **String theory**

Symmetry broken to O(d,d;Z), contains GL(d,Z)

Exact **T-duality** symmetry of string theory

Includes strange non-geometric transformations

Gravity + C-field+...

Effective theory on N: Symmetry E_{d+1}

Full quantum theory: **M-theory**

$E_{d+1}(Z)$ Exact **U-duality** symmetry of M-theory

T-Duality

$$X = X_L(\sigma + \tau) + X_R(\sigma - \tau), \qquad \tilde{X} = X_L - X_R$$

X conjugate to momentum, $\tilde{X}$ to winding no.

2-d version of electromagnetic duality $\quad X \to \widetilde{X}$

$$dX = *d\widetilde{X} \qquad \partial_a X = \epsilon_{ab}\partial^b \widetilde{X}$$

$$R \to \widetilde{R} = 1/R, \qquad n \to \widetilde{n} = m,\ m \to \widetilde{m} = n$$

T-Duality

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Fundamental quanta $\longleftrightarrow$ Solitons

Momentum $\longleftrightarrow$ Winding

New Symmetry of physics: large R $\longleftrightarrow$ small R

Strings on d-Torus

Target space $T^d \times \mathbb{R}^D$ T^d Coordinates $X^i, i = 1, ..., d$

Moduli on torus (constant) $G_{ij},\ B_{ij}$ $E_{ij} = G_{ij} + B_{ij}$

T-Duality Symmetry $O(d,d;\mathbb{Z})$

i) Large Diffeos $GL(d;\mathbb{Z})$

ii)B-shifts $B \to B + \Theta, \quad \Theta_{ij} \in \mathbb{Z}$

iii) Inversions $R_i \to 1/R_i$

$$E \to (aE+b)(cE+d)^{-1} \qquad h = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(d,d;Z)$$

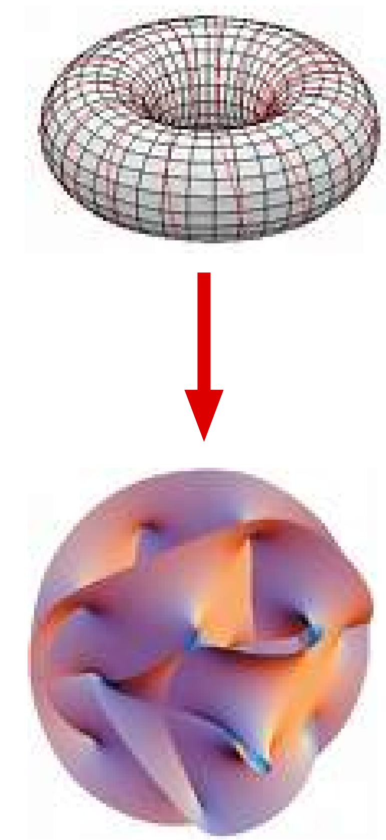
$\left|p_i, w^i\right\rangle$ Lie in 2d-lattice, action of $O(d,d;\mathbb{Z})$

T-Duality

$G(Y), B(Y)$

X^i

Y^m



- Space has d-torus fibration
- G,B on fibres
- T-Duality O(d,d;Z), mixes G,B
- Mixes Momentum and Winding
- Changes geometry and topology

$$E \to (aE+b)(cE+d)^{-1}$$

$$h = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(d,d;Z) \qquad E_{ij} = G_{ij} + B_{ij}$$

On circle, radius R: $\quad O(1,1;\mathbb{Z}) = \mathbb{Z}_2 : R \mapsto \dfrac{1}{R}$

Moduli Space of CFT's

$$\frac{O(d,d)}{O(d)\times O(d)}$$ Identified under $O(d,d;\mathbb{Z})$

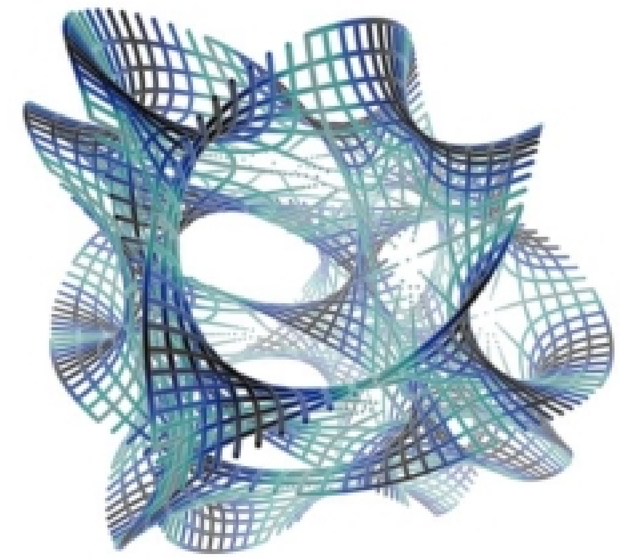
d^2 Moduli: E=G+B

T-duality map between equivalent representations of same CFT.

Non-Geometric String Backgrounds

Geometric Background

Manifold with tensor fields, fluxes and gauge fields



Duality ⇊

Non-Geometric Background ???

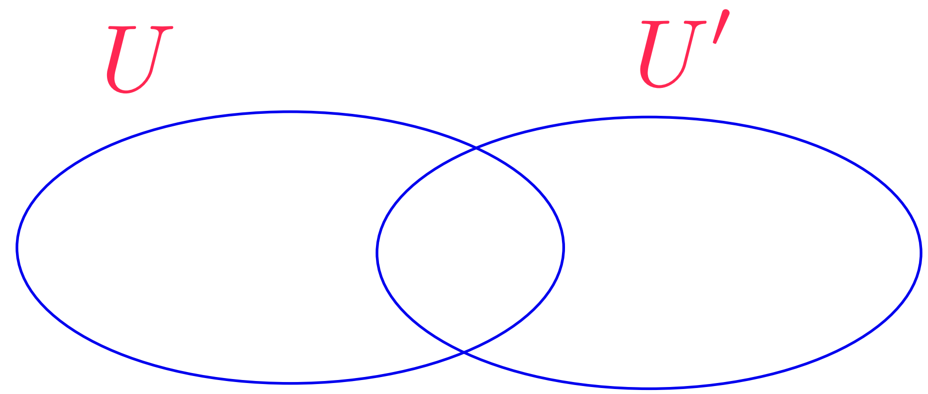
Dualities: stringy symmetries

Usually maps to another geometric background

But sometimes not:

Obstruction to duality?

Or non-geometric background?

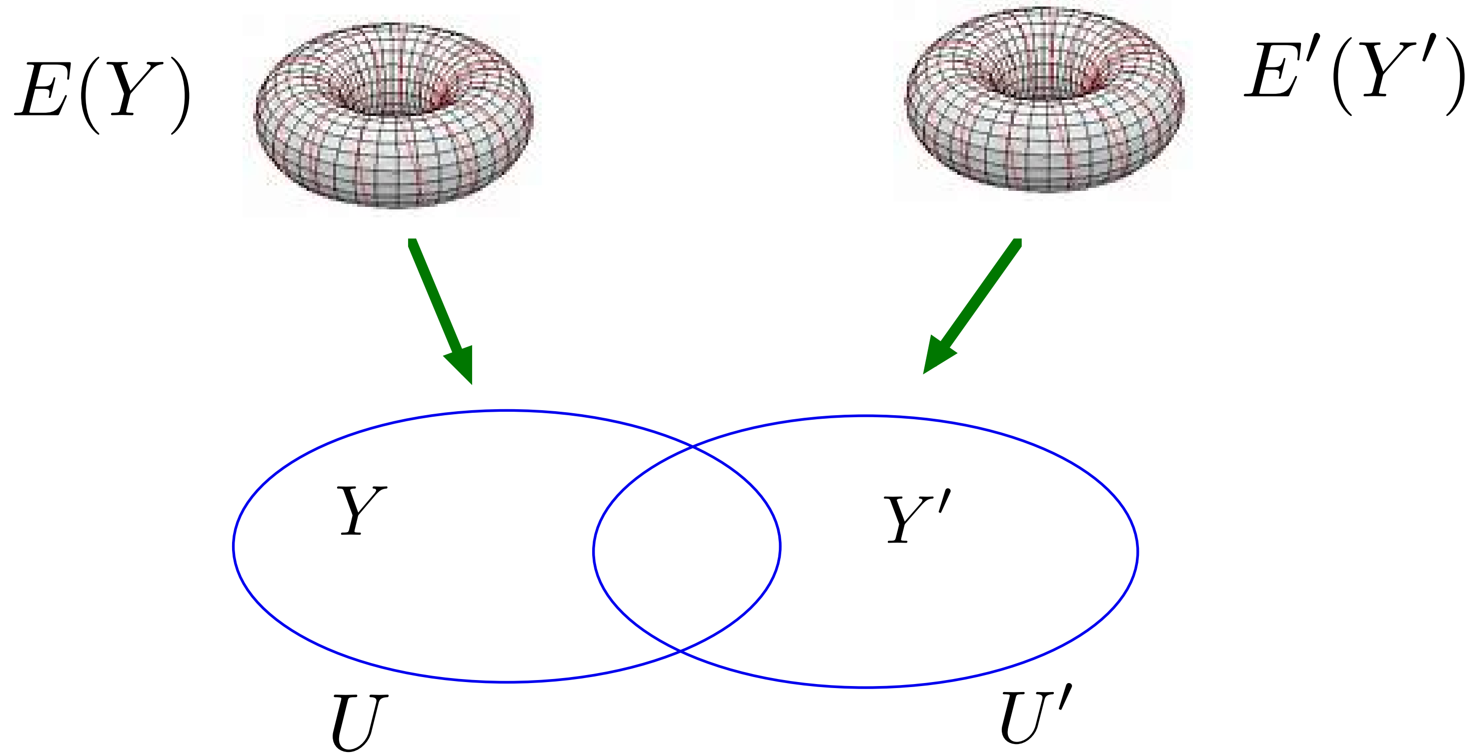


Patches glued using geometric symmetries:
Diffeomorphisms, gauge transformations
to construct geometric background

If toroidal fibration:

$\Big\downarrow$ T-duality

Glue using T-dualities also
T-fold: Patching uses T-duality
Physics smooth, as T-duality a symmetry



Torus fibration

Geometric background: G, H=dB tensorial

T-fold: Transition functions involve T-dualities (as well as diffeomorphisms and 2-form gauge transformations)

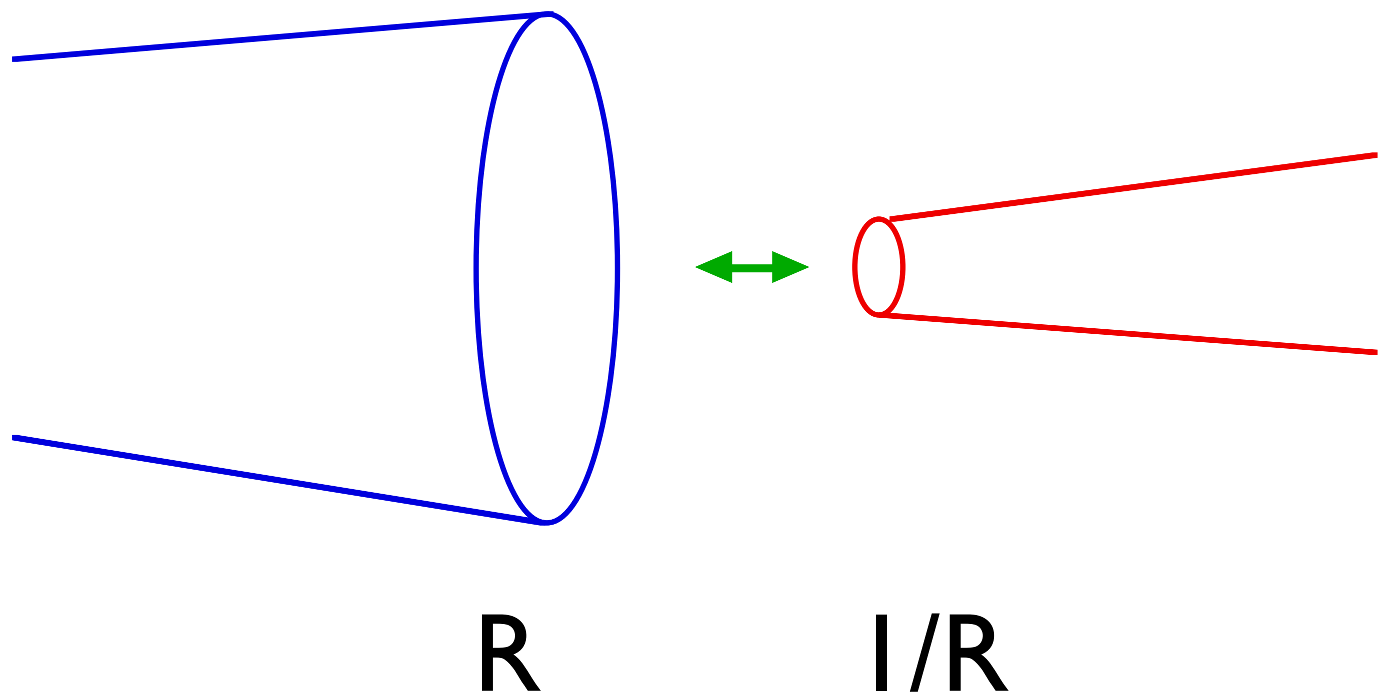
E=G+B Non-tensorial

$O(d,d;\mathbb{Z})$ $\qquad E' = (aE+b)(cE+d)^{-1} \;\text{ in }\; U \cap U'$

Glue using T-dualities also → **T-fold**

Physics smooth, as T-duality a symmetry

T-fold patching



Glue big circle (R) to small (1/R)
Glue momentum modes to winding modes
(or linear combination of momentum and winding)
Not conventional smooth geometry

Non-Geometric Backgrounds

Many consistent non-geometric string backgrounds
Orbifolds, asymmetric orbifolds:
    arise in NGB at special points in moduli space
T-folds, U-folds
    spaces with torus fibration and T or U duality patching,
Mirror-folds
    with CY fibration and mirror symmetry patching

T-Duality Monodromies Round Degenerate Fibres

S. Hellerman, J. McGreevy and B. Williams 2002

Torus fibration over base
Singularities in base where fibre degenerates
T-duality monodromies round singularities

Reductions with Duality Twist

A. Dabholkar and C. Hull 2002

Compactify on circle (or torus)
T-duality or U-duality Monodromy round circle(s)
Stringy lift of Scherk-Schwarz
At special points in mod space: assym orbifolds

Strings on T^d

$$X = X_L(\sigma + \tau) + X_R(\sigma - \tau), \qquad \tilde{X} = X_L - X_R$$

X conjugate to momentum, $\tilde{X}$ to winding no.

$$dX = *d\tilde{X} \qquad \partial_a X = \epsilon_{ab}\partial^b \tilde{X}$$

Need "auxiliary" $\tilde{X}$ for interacting theory

i) Vertex operators $e^{ik_L \cdot X_L}, \quad e^{ik_R \cdot X_R}$

ii) String field Kugo & Zwiebach $\Phi[x, \tilde{x}, a, \tilde{a}]$

Strings on T^d

$$X = X_L(\sigma+\tau) + X_R(\sigma-\tau), \qquad \tilde{X} = X_L - X_R$$

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Doubled Torus 2d coordinates

Transform linearly under $O(d,d;\mathbb{Z})$

Doubled sigma model CMH 0406102

$$X \equiv \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix}$$

Doubled Geometry for T-fold

Hull

$$\mathbb{X}^I = \begin{pmatrix} X^i \\ \widetilde{X}_i \end{pmatrix} \qquad I = 1, ..., 2d$$

Transforms linearly under $O(d, d; \mathbb{Z})$

T-fold transition: mixes $X, \tilde{X}$

No global way of separating "real" space coordinate X from "auxiliary" $\widetilde{X}$

Duality covariant formulation in terms of $\mathbb{X}$

Transition functions $O(d, d; \mathbb{Z}) \subset GL(2d; \mathbb{Z})$

can be used to construct bundle with fibres T^{2d}

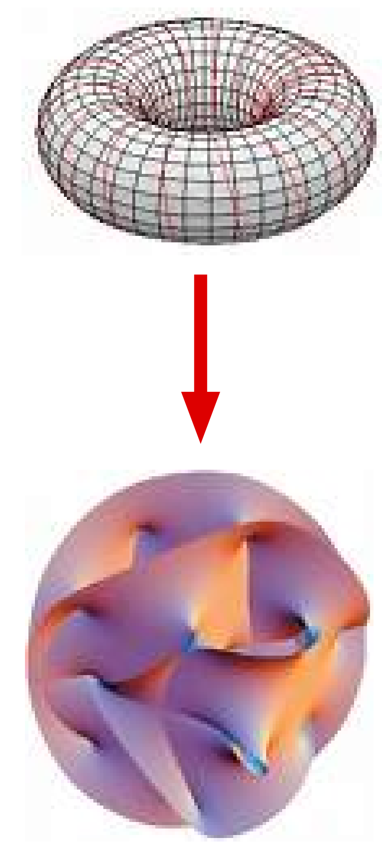
Doubled space is smooth manifold!

Sigma Model on doubled space

Doubled Bundle

T^{2d} bundle: doubled fibre
Construct duality-covariant sigma model on doubled space $(\mathbb{X}^I, Y^m)$
Constraint to halve degrees of freedom on fibre:

$\mathbb{X}^I$



Y^m

$$dX = * d\widetilde{X}$$ for free case

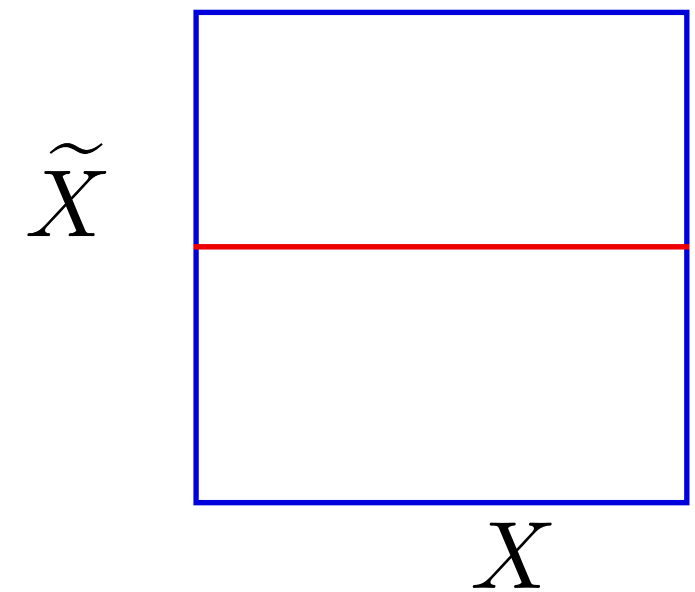
$$D\mathbb{X} = S(Y) * D\mathbb{X}$$ for general case

$$S(Y) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + O(Y) \qquad S^2 = 1$$

Polarisation

To recover conventional formulation, split into "fundamental" and "auxiliary":

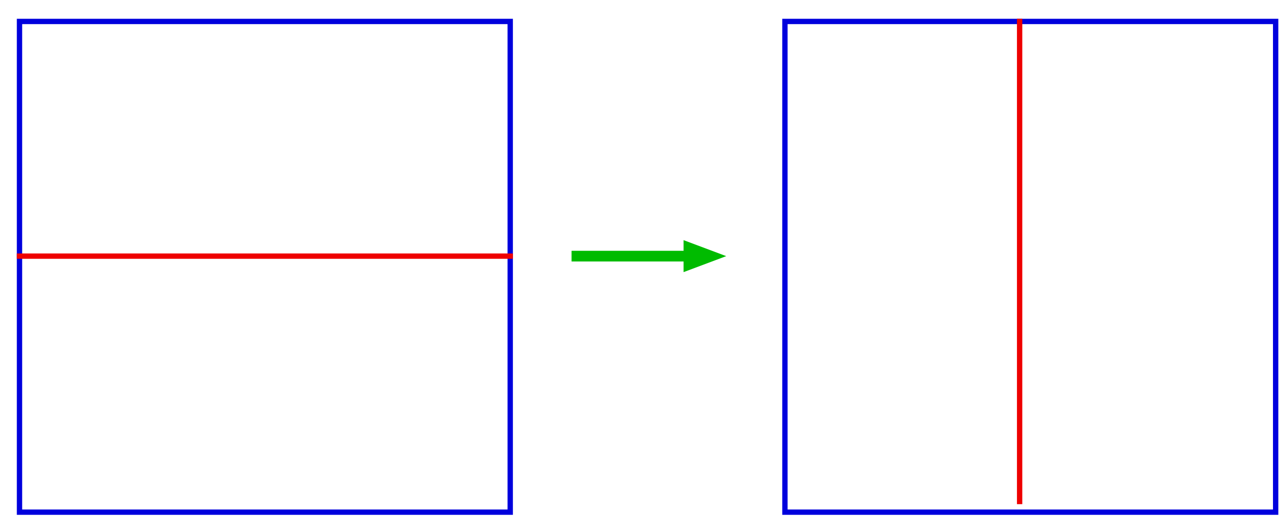
$$\mathbb{X} \to \{X^i, \tilde{X}_i\}$$



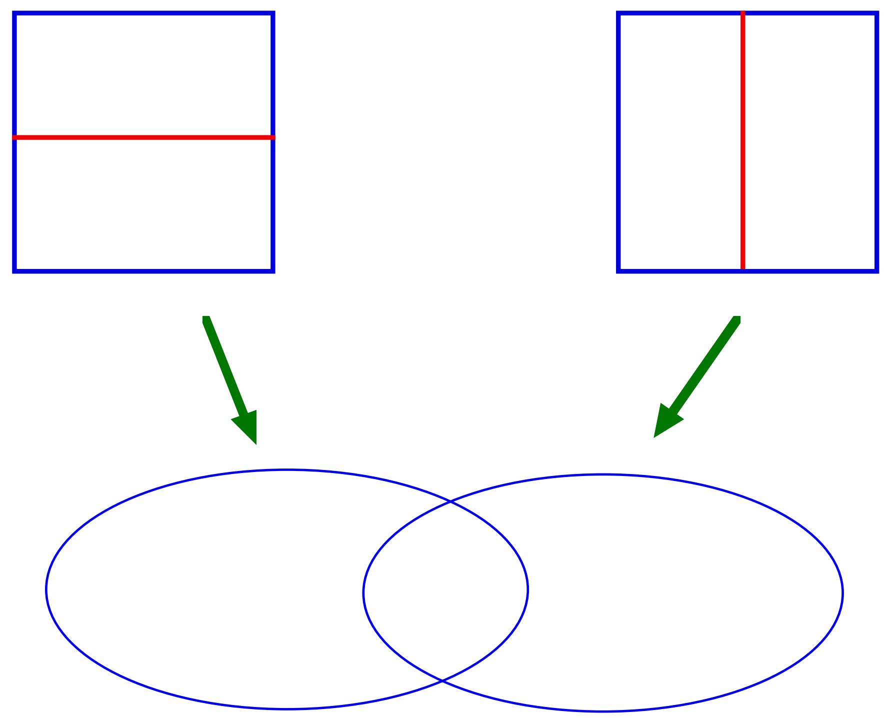
Pick "real spacetime", $T^d \subset T^{2d}$

T-duality rotates polarisation.

T-duality symmetry: physics independent of polarisation.

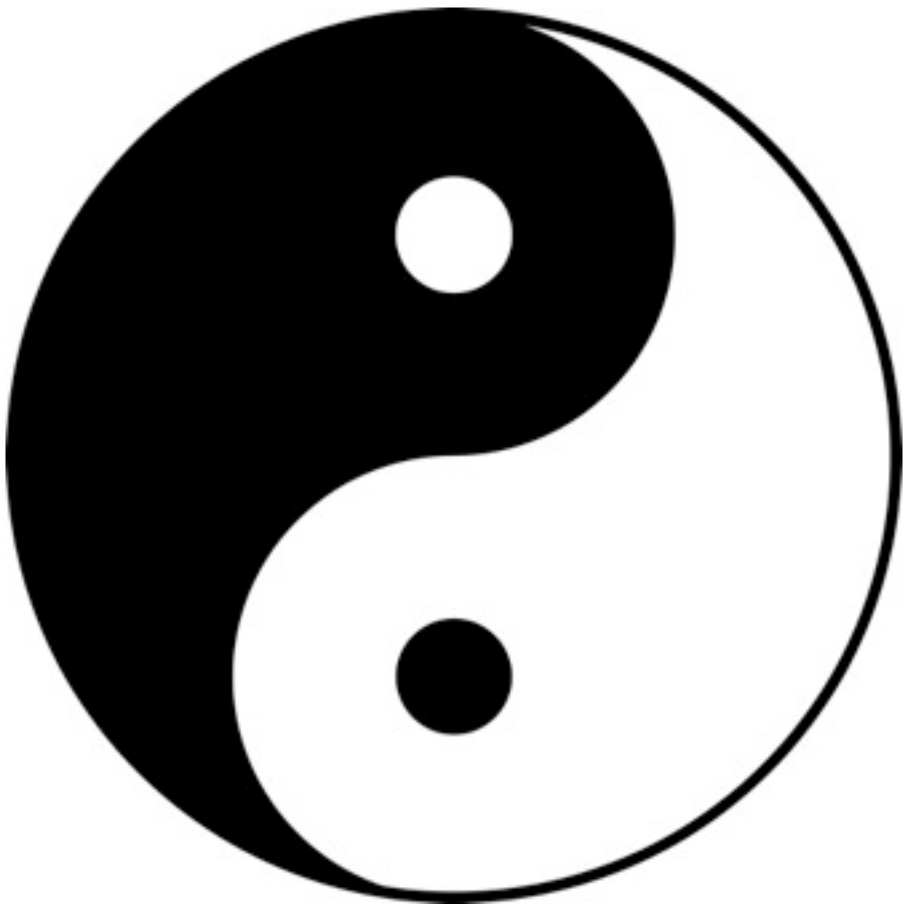


T-fold



Pick polarisation over each patch in base.
T-duality transitions: polarisation changes from patch to patch.
Geometric: there is global spacetime submanifold
Non-geometric if there is no global polarisation.

Double Field Theory



CMH & Barton Zwiebach arXiv:0904.4664, 0908.1792
CMH & BZ & Olaf Hohm arXiv:1003.5027, 1006.4664

String Theory on a Torus

Infinite set of double fields

$$g_{ij}(x,\tilde{x}), b_{ij}(x,\tilde{x}), \phi(x,\tilde{x}), \ldots, C_{ijk\ldots l}(x,\tilde{x}), \ldots$$

- Fields on double space satisfy differential constraint
- T-duality symmetry
- Coordinates doubled, but tensor indices not
- Each field carries "level numbers" $N, \bar{N}$

Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

Treat as field equation, kinetic operator in doubled space

$$G^{ij}\frac{\partial^2}{\partial x^i \partial x^j} + G_{ij}\frac{\partial^2}{\partial \tilde{x}_i \partial \tilde{x}_j}$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Treat as constraint on double fields

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} \qquad (\Delta - \mu)\psi = 0$$

Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

Treat as field equation, kinetic operator in doubled space

$$G^{ij} \frac{\partial^2}{\partial x^i \partial x^j} + G_{ij} \frac{\partial^2}{\partial \tilde{x}_i \partial \tilde{x}_j}$$

Laplacian for metric

$$ds^2 = G_{ij} dx^i dx^j + G^{ij} d\tilde{x}_i d\tilde{x}_j$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Treat as constraint on double fields

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} \qquad (\Delta - \mu)\psi = 0$$

Laplacian for metric

$$ds^2 = dx^i d\tilde{x}_i$$

$$g_{ij}(x,\tilde{x}), b_{ij}(x,\tilde{x}), \phi(x,\tilde{x})$$

$$N = \bar{N} = 1$$

$$p^2 + w^2 = 0$$

$$p \cdot w = 0$$

"Double Massless"

Torus Backgrounds

$$G_{ij} = \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & G_{ab} \end{pmatrix}, \quad B_{ij} = \begin{pmatrix} 0 & 0 \\ 0 & B_{ab} \end{pmatrix} \qquad E_{ij} \equiv G_{ij} + B_{ij}$$

Fluctuations $\quad e_{ij} = h_{ij} + b_{ij}$

Take $\quad B_{ij} = 0 \qquad \tilde{\partial}_i \equiv G_{ik} \dfrac{\partial}{\partial \tilde{x}_k}$

Torus Backgrounds

$$G_{ij} = \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & G_{ab} \end{pmatrix}, \quad B_{ij} = \begin{pmatrix} 0 & 0 \\ 0 & B_{ab} \end{pmatrix} \qquad E_{ij} \equiv G_{ij} + B_{ij}$$

Fluctuations $\quad e_{ij} = h_{ij} + b_{ij}$

Take $\quad B_{ij} = 0 \qquad \tilde{\partial}_i \equiv G_{ik} \dfrac{\partial}{\partial \tilde{x}_k}$

Usual action $\quad \displaystyle\int dx \sqrt{-g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12} H^2 \right]$

Quadratic part $\quad \displaystyle\int dx \; L[\, h, b, d; \partial \,]$

$e^{-2d} = e^{-2\phi} \sqrt{-g}$ (d invariant under usual T-duality)

Double Field Theory Action

$$S^{(2)} = \int [dx d\tilde{x}] \; \Big[\; L[\, h, b, d; \partial \,] + L[\, -h, -b, d; \tilde{\partial} \,]$$

$$+ \, (\partial_k h^{ik})(\tilde{\partial}^j b_{ij}) + (\tilde{\partial}^k h_{ik})(\partial_j b^{ij}) - 4\, d\, \partial^i \tilde{\partial}^j b_{ij} \; \Big]$$

Action + dual action + strange mixing terms

Free Double Field Theory Action

$$S^{(2)} = \int [dx d\tilde{x}] \Big[\; L[\, h, b, d; \partial\,] + L[\,-h, -b, d; \tilde{\partial}\,]$$

$$+ (\partial_k h^{ik})(\tilde{\partial}^j b_{ij}) + (\tilde{\partial}^k h_{ik})(\partial_j b^{ij}) - 4\, d\, \partial^i \tilde{\partial}^j b_{ij} \; \Big]$$

Action + dual action + strange mixing terms

$$\delta h_{ij} = \quad \partial_i \epsilon_j \; + \partial_j \epsilon_i \; + \; \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i \; ,$$

$$\delta b_{ij} = -(\tilde{\partial}_i \epsilon_j - \tilde{\partial}_j \epsilon_i) - (\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i)\, ,$$

$$\delta d \; = -\, \partial \cdot \epsilon + \; \tilde{\partial} \cdot \tilde{\epsilon}\, .$$

Invariance needs constraint

Diffeos and B-field transformations mixed.

Invariant cubic action found for full DFT of (h,b,d)

Constrained fields $\psi(x^\mu, x^a, \tilde{x}_a)$

$$(\Delta - \mu)\psi = 0$$

Momentum space $\psi(p_\mu, p_a, w^a)$ $\Delta = p_a w^a$

Momentum space: Dimension n+2d
Cone: $p_a w^a = 0$ or hyperboloid: $p_a w^a = \mu$
dimension n+2d-1

DFT: fields on cone or hyperboloid, with discrete p,w
Problem: naive product of fields on cone do not lie on cone. Vertices need projectors

Restricted fields: Fields that depend on d of 2d torus momenta, e.g. $\psi(p_\mu, p_a)$ or $\psi(p_\mu, w^a)$
Simple subsector, no projectors needed, no cocycles.