"Infinite-dimensional Structures in Higher Geometry and Representation Theory"

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Abstracts

Beltita, Daniel "*Operator valued Fourier transforms on nilpotent Lie groups*"

We will discuss the image of operator valued Fourier transforms on general nilpotent Lie groups, following the pattern from Heisenberg groups. To this end we use the method of coadjoint orbits and stratifications of the duals of nilpotent Lie algebras. We thus show that the C^*-algebra of any nilpotent Lie group is a solvable C^*-algebra, that is, it admits a finite increasing sequence of closed two-sided ideals whose successive quotients are isomorphic to algebras of compact-operator valued continuous functions that vanish at infinity on suitable locally compact spaces. This is an improvement of a result of N.V. Pedersen (1984), who established the existence of such finite composition series whose successive quotients were C^*-algebras with continuous trace. In addition, a characterization of Heisenberg groups in terms of group C^*-algebras will be provided along these lines. The talk is based on joint work with Ingrid Beltita and Jean Ludwig.

Carey, Alan “*Invariants from KK theory*”

This talk will provide a brief overview of Kasparov’s bivariant KK theory. I will link this to approaches to the problem of detecting torsion in K-theory using the Kasparov product and recent progress in explicit constructions of unbounded Kasparov modules. These developments were partly motivated by twisted K-theory and uses of K-theory in the study of topological insulators.

Ganter, Nora “*Representations of categorical tori*”

Categorical groups (also known under the name 2-groups) are monoidal category in which the objects are weakly invertible. It turns out that many groups that we know and love turn up in nature as categorical groups, and the study of their previously ignored categorical nature reorganizes and often greatly simplifies our understanding of these objects. There is strong evidence that categorical groups provide a “geometric” pendant to affine representation theory that avoids many of the technicalities involved in loop group representations.

Examples of categorical groups include categorical tori, the string 2-groups, the Platonic 2-groups, and, conjecturally, the Refined Monster. This talk will focus on a very simple example, namely the categorical tori and their representations. I will explain what they are and relate the basic representation of the categorical Leech torus to the Refined Monster.

Glöckner, Helge “*Infinite-dimensional calculus with a view towards Lie theory*”

The talk provides an introduction to calculus in locally convex spaces, with an emphasis on techniques which are useful in the theory of infinite-dimensional Lie groups. Manifolds modelled on locally convex spaces will be introduced and examples of Lie groups modelled on locally convex spaces will be given. The techniques discussed subsume exponential laws, implicit function theorems, and direct limit constructions. The talk closes with a discussion of regularity properties of infinite-dimensional Lie groups, including very recent results on strengthened regularity properties like C^k-regularity and L^p-regularity.

Grundling, Hendrik "*Crossed products of C^*-algebras for singular actions*"
We consider group actions $\alpha\colon G \to \mathrm{Aut}(\mathcal{A})$ of topological groups G on C^*-algebras $\mathcal{A}$ of the type which occur in many quantum physics models. These are singular actions in the sense that they need not be strongly continuous, or the group need not be locally compact. We develop a "crossed product host" in analogy to the usual crossed product for strongly continuous actions of locally compact groups, in the sense that its representation theory is in a natural bijection with the covariant representation theory of the action $\alpha\colon G \to \mathrm{Aut}(\mathcal{A})$. We have a uniqueness theorem for crossed product hosts, useful existence conditions, and a number of examples where a crossed product host exists, but the usual crossed product does not.

In the case that the class of permissible covariant representations is restricted by a positive spectral condition, there is additional theory available for construction of crossed product hosts, and if time permits, we will discuss some results in this area.

Hanisch, Florian "*Infinite-dimensional supergeometry, mapping spaces and applications*"
Supergeometry is the geometry of spaces whose rings of functions contain commuting as well as anticommuting elements. There are different approaches to the subject, we will briefly discuss the ringed space picture and the functorial approach. In contrast to the ringed space approach, the functorial one is directly meaningful for the definition of infinite-dimensional objects. We will discuss the construction of "mapping space"-objects within this framework. These are functors from the category of Grassmann algebras into a suitable category of manifolds, defined by an "exponential-law-type" formula. We will first describe their algebraic properties and then discuss the existence of smooth (or, more precisely, supersmooth) structures, i.e. the existence of appropriate atlases on these functors. It turns out that in the case of non-compact domains (which is particular important in view of physical applications), additional difficulties arise due to a certain incompatibility of the standard construction of mapping spaces on the one hand and functoriality requirements on the other hand. We suggest a possible (though not entirely satisfactory) solution to this problem by slightly enlarging the model spaces under consideration. If time allows, we will

briefly discuss some applications.

Hekmati, Pedram "*String Structures, Reductions and T-duality*"

In this talk I will explain the relation between string structures and reduction of Courant algebroids. I will describe how these structures behave under topological T-duality and discuss some of its implications. This is joint work with David Baraglia.

Janssens, Bas "*A fusion structure for the spinor bundle on loop space*"

There is a canonical way to assign to each loop on a Riemannian manifold a Hilbert space with a conformal net that acts on it. The failure of these Hilbert spaces to constitute a locally trivial bundle over loop space is measured by the curvature of the metric. We use higher geometry (2-groups and 2-bundles) to describe this situation and outline why the Connes fusion product of Hilbert spaces should extend to a fusion structure for the spinor bundle on loop space. This is work in progress.

Karshon, Yael "*An Invitation to Diffeology*"

Diffeology, introduced around 1980 by Jean-Marie Souriau following earlier work of Kuo-Tsai Chen, gives a simple way to extend notions of differential topology beyond manifolds. A diffeology on a set specifies which maps from open subsets of Euclidean spaces to the set are "smooth". Examples include (possibly non-Hausdorff) quotients of manifolds and (infinite dimensional) spaces of smooth mappings between manifolds. I will present some examples and results that relate diffeology with more traditional aspects of Lie group actions.

Mickelsson, Jouko "*Transgression of gauge group cocycles*"

Denoting by $\mathcal{G}_n$ the group of based smooth maps $g\colon S^n \to G$ where G is a compact Lie group we have a natural fibration $\mathcal{G}_n \to A \to \mathcal{G}_{n-1}$ if $\mathcal{G}_{n-1}$ is connected, with A a contractible space. Under certain restrictions on the homology groups of $\mathcal{G}_n$ one can construct locally smooth groupoid cocycles in a geometric way for the transformation groupoid $A \times \mathcal{G}_n \to A$. We address the question when a groupoid k-cocycle of $\mathcal{G}_n$ transgresses to a (locally smooth) groupoid $(k+1)$-cocycle of $\mathcal{G}_{n-1}$. This can be viewed as a global formulation corresponding to the infinitesimal (Lie algebra) transgression arising from the BRS double complex in gauge theory. In particular, the case $k = 2$ is of interest, related to gerbes on a group manifold and the categorical approach to gauge group representation theory. (This talk is part of a work "Third group cohomology and gerbes over Lie groups" in progress with Stefan Wagner.)

Pronk, Dorette "*Mapping Spaces for Orbispaces*"

Orbifolds were originally introduced by Satake in 1956 under the name V-manifolds and further popularized by Thurston under the name orbifolds. For this talk I won't be interested in the smooth structure so I will work with *orbispaces*: an orbispace is given by an underlying paracompact Hausdorff space with an atlas of charts that gives it locally the structure of a quotient of Euclidean space by the action of a finite group.

One way to model orbispaces is by étale topological groupoids with a proper diagonal (groupoids internal to the category of topological spaces with étale structure maps). We will call such groupoids *orbigroupoids*.

One way to define a notion of maps between orbispaces is start with the category of internal groupoids and continuous groupoid homomorphisms and invert the essential equivalences (the internal version of a weak equivalence of categories) by taking the bicategory of fractions with respect to the essential equivalences. The resulting maps between orbispaces have been called *good maps* or *generalized maps*. They also correspond to Hilsum-Skandalis maps between the groupoids (which are defined in terms of bimodules.) For this talk we will call them *orbimaps*.

For orbigroupoids $\mathcal{G}$ and $\mathcal{H}$ where $\mathcal{G}$ is compact in the sense that the quotient space is compact, we will study the structure of the mapping groupoid

OrbiMaps$(\mathcal{G}, \mathcal{H})$. We will show that this can be described as the pseudo colimit of a diagram of mapping groupoids **GpdMaps**$(\mathcal{G}', \mathcal{H})$ in the 2-category of topological groupoids for a set of orbigroupoids $\mathcal{G}'$ with an essential equivalence $\mathcal{G}' \to \mathcal{G}$. This ensures that the resulting groupoid is small. We will then further discuss the local topological structure and some key properties that are needed to work with these groupoids and in particular show that they are again étale and proper, i.e., orbigroupoids.

This is joint work with Laura Scull (Fort Lewis College, USA)

Roberts, David "*A new String group model from LG*"

Known models for the string group are generally built using the central extension of the based loop group in some way. One also has that the loop group that is used may have use loops with a non-smooth point at the origin. This is at odds with the idea that the group LG of free loops in a Lie group G, together with its S^1-action by rotation of loops is important for string-theoretic considerations (one is mindful the Witten genus, for instance). This talk will cover some background on String groups, and give a new construction involving LG, arising in joint work with Murray and Wockel. This model is coherent, rather than strict, and in fact we show it cannot be a strict Lie 2-group. This model is thus slightly more complicated that that of Baez-Stevenson-Crans-Schreiber (a strict 2-group), but less complicated that that of Schommer-Pries (a Lie groupoid with multiplication given by a bibundle).

Salmasian, Hadi "*Spherical polynomials and the spectrum of invariant differential operators for the supersymmetric pair $GL(m, 2n)/OSp(m, 2n)$*"

The algebra of invariant differential operators on a multiplicity-free representation of a reductive group has a concrete basis, usually referred to as the Capelli basis. The spectrum of the Capelli basis on spherical representations results in a family of symmetric polynomials (after ρ-shift) which has been studied extensively by Knop and Sahi since the early 90's. In this talk, we generalize some of the Knop-Sahi results to the symmetric superpair $GL(m, 2n)/OSp(m, 2n)$. As a side result, we show that the qualitative Capelli problem (in the sense of Howe-Umeda) for this superpair has an affirmative answer. Finally, we prove that in the Frobenius coordinates of Sergeev-Veselov, our polynomials turn into the shifted super Jack polynomials. This talk is based on an ongoing project with Siddhartha Sahi.

Schmeding, Alexander "*The Lie group of bisections of a Lie groupoid*"

In the talk we give a short introduction to bisections of Lie groupoids together with some elementary examples. One may think of a bisection as a "generalised element" of a groupoid. It is well known that the bisections associated to a fixed groupoid form a group. We will outline how the group of bisections can be endowed with a natural locally convex Lie group structure if the Lie groupoid has compact space of objects. Moreover, we develop the connection to the algebra of sections of the associated Lie algebroid and discuss the implications of this construction on the level of Lie functors between suitable categories. This is joint work with Christoph Wockel (Universität Hamburg)

Vozzo, Raymond "*String structures on homogeneous spaces*"

In this talk I will describe some examples of string structures on homogeneous spaces. Waldorf has given a definition of string structures, which includes connections, as certain trivialisations of the Chern–Simons bundle 2-gerbe. I will give explicit examples of such trivialisations and explain some associated work illustrating how such structures are related to equivariant bundle gerbes. I will also report on some work in progress describing the connective structure on these objects. This is joint work with D. Roberts, M. Murray, and D. Stevenson.

Wagemann, Friedrich "*On the string Lie algebra*"

This is joint work with Salim Rivière. We interprete the equivalence class of crossed modules corresponding to the Cartan cocycle for a semi-simple finite dimensional complex Lie algebra as the "string Lie algebra". We exhibit in this equivalence class a so-called "abelian representative" which is meant to simplify applications of the string Lie algebra in representation theory. As an illustration, we show that Cirio-Martins' construction of a categorification of the infinitesimal braiding in a category of modules corresponding to the string Lie algebra for $\mathfrak{sl}_2$ extends to all finite dimensional metric Lie algebras.

Wagner, Stefan "*Free actions on C*-algebras and (group -) cohomology*"

The classification of group actions on C*-algebras is intrinsically interesting and the experience with the commutative case suggests that free group actions are easier to understand and to classify than general actions. Being global in nature, the strategy in the C*-algebraic setting is to classify free group actions on C*-algebras in terms of their associated isotypic components. In fact, the classification process incorporates the theory of imprimitivity bimodules and methods from group cohomology. The applications we have in mind involve the development of a fundamental group for C*-algebras and an approach to "quantum gerbes". This is joint work with Kay Schwieger.

Waldorf, Konrad "*Transgressive central extensions of loop groups*"

Some central extensions of the loop group LG of a Lie group G can be obtained as the transgression of a gerbe over G. I will describe a loop group-theoretical characterization of such central extensions in terms of loop fusion and thin homotopy equivariance. As examples, I will discuss the universal central extension of a compact simply-connected Lie group, which is transgressive, and several extensions of $LU(1)$, of which some are transgressive and others not.

Zhu, Chenchang "*Higher structures in differential geometry*"

We will give an overview of higher structures in differential geometry (higher groupoids, higher stacks, various differential category including infinite-dimensional ones) and in particular what they are good for. Then we will explain how to organize these objects in a reasonable way to work with. For this we will establish a higher category for higher groupoids in various pretopologies, including higher (Banach) Lie groupoids. To build a higher category for these objects, one convenient method is to build a category of fibrant objects for them. For this, we need to choose path objects, weak equivalences, and fibrations. It turns out that our choice of weak equivalence, restricting to Lie (1-)groupoids, are the well known weak equivalence for Lie groupoids. Thus the (higher) category built for Lie (1-)groupoids will be the 2-category of Lie groupoids which is equivalent to that of differential stacks. Many of the ideas above are well known to experts in the field, and the talk itself is based on work in progress with Chris Rogers.

In the end, we will give one more example where these higher infinite-dimensional Lie groups are used, that is in the integration problem of infinite-dimsional Lie algebras. This is a joint work with Christoph Wockel.