

## Exercise Sheet 6

### Transformation Groups

**Exercise 1:** Let  $G$  be a compact Lie group and  $H$  a closed subgroup of  $G$ . Show that there exists a representation  $V$  of  $G$  and a point  $v \in V$  with  $G_v = H$ . Hint: Use 4.7.3 and the proof of 4.7.5.

**Exercise 2:** Let  $G$  be a compact Lie group and  $H$  a closed subgroup. Let  $U$  be any neighbourhood of  $e$ . Show that there is a neighbourhood  $W \subseteq U$  of  $e$  such that, whenever  $K$  is a subgroup contained in  $WH$ , then  $K$  is subconjugate to  $H$  via an element of  $U$ , i.e.  $uKu^{-1} \subseteq H$  for some  $u \in U$ . Hint: Action of the Week.

**Exercise 3:** Let  $G$  be a compact Lie group and  $K \subseteq H$  subgroups. The normalizer  $N(K)$  acts on the fixed space  $(G/H)^K$ . Use Exercise 2 to show that the orbit space of this action is finite.

**Handing in:** 02.02.12 in exercise class.

## Action of the Week

Group: Any Lie group  $G$

Space:  $X(H) = \{A \mid A \subseteq G/H \text{ closed}\}$

Action:  $G \times X(H) \rightarrow X(H), (g, A) \mapsto gA$

Isotropies:

Further properties:  $X(H)$  is a metric space with the Hausdorff metric induced from  $G$  (which is a manifold, hence metric)