Dr. Philipp Wruck 09.01.2012

Exercise Sheet 5
Transformation Groups

Exercise 1: Consider the Lie group

$$G = SL(2, \mathbb{R}) = \{A \in L(\mathbb{R}^2, \mathbb{R}^2) \mid \det A = 1\}.$$

1. Calculate the tangential space of G at e.
2. Show that the exponential map $\exp : T_eG \to G$ is not surjective and describe its image.

Exercise 2: Let G be any Lie group. Prove that there is a neighbourhood U of e in G such that $\{e\}$ is the only subgroup of G contained in U.

Exercise 3: Let M be a G-manifold with G a compact Hausdorff topological group. Let U be a neighbourhood of an orbit Gx, $x \in M$. Show that U contains an invariant neighbourhood of Gx.

Handing in: 19.01.12 in exercise class.

Action of the Week

Group:	Any Lie group G
Space:	TM for a smooth G-manifold M
Action:	For $v \in T_xM$, $g.v = T_xg(v) \in T_{gx}M$
Isotropies:	$G_v \subseteq G_x$
Further properties:	Determines an action of G on the bundle $TM \to M$