Dr. Philipp Wruck 08.12.2011

Exercise Sheet 4
Transformation Groups

Exercise 1: Let $p_n : \mathbb{S}^1 \to \mathbb{S}^1,\ z \mapsto z^n$ for $n \in \mathbb{N}$.

1. Show that p_n is a $\mathbb{Z}_n$-principal bundle.
2. Consider the associated bundle $\pi : \mathbb{S}^1 \times_{\mathbb{Z}_n} \mathbb{S}^1 \to \mathbb{S}^1$, where $\mathbb{Z}_n$ acts on $\mathbb{S}^1$ canonically. Show that π is a $\mathbb{S}^1$-principal bundle.
3. Show that π is trivial as an $\mathbb{S}^1$-principal bundle, but not trivial as a fibre bundle with structure group $\mathbb{Z}_n$.

Exercise 2: Let $p : E \to B$ be a principal K-bundle and Y a left K-space. Considering Y as a right K-space via $y.k = k^{-1}.y$, show that there is a bijection between the set $\mathcal{TOP}_K(E, Y)$ and the set of cross sections of the Y-bundle

$$\pi : Y \times_K E \to B$$

given by sending a section $\sigma : B \to Y \times_K E$ to the map $f : E \to Y$ such that $\sigma \circ p(x) = [f(x), x]$.

Exercise 3: Let $p : E \to B$, $p' : E' \to B'$ be two fibre bundles with compact fibre and $(F, f) : p \to p'$ a bundle map. Show that (F, f) is an isomorphism of fibre bundles if and only if f is a homeomorphism and F is a homeomorphism restricted to fibres.

Handing in: 22.12.11 in exercise class.

Action of the Week

Group:	Any group G
Space:	$\mathcal{TOP}(X, Y)$ for G-spaces X, Y
Action:	$(g.f)(x) = g^{-1}.f(g.x)$
Isotropies:	Any closed subgroup of G
Further properties:	$\mathcal{TOP}(X, Y)^H$ is the space of H-equivariant maps $X \to Y$