Dr. Philipp Wruck 15.11.2011

Exercise Sheet 2
Transformation Groups

Exercise 1: Let G be a connected topological group and U a neighbourhood of e. Show that $G = \bigcup_{n\geq 1} U^n$. So every neighbourhood of the unit element generates the whole group.

Exercise 2: Let G be a compact Hausdorff topological group and $H \subseteq G$ a closed subgroup. Show that the canonical map

$$N(H)/_{H} \to Homeo_G(G/_{H}),\ [n] \mapsto ([g] \mapsto [gn])$$

is a homeomorphism (here $N(H)$ is the normalizer of H and mapping spaces carry the CO-topology).

Exercise 3: The special orthogonal group $SO(n)$ acts on the space of symmetric matrices of trace 0 via conjugation.

(a) Determine the orbits of this action.

(b) Determine the isotropy subgroups of this action.

(c) Show that the quotient space can be identified with a cone in $\mathbb{R}^{n-1}$.

Handing in: 24.11.11 in exercise class.

Action of the Week

Group:	S_n
Space:	X^n for any topological space X
Action:	$(\sigma, x) \mapsto (x_{\sigma(1)}, \ldots, x_{\sigma(n)})$
Isotropies:	$\prod_{i=1}^{k} S_{j_i}$ with $\sum_{i=1}^{k} j_i = n$
Further properties:	Quotient called n-fold symmetric product