Exercises for Higher Structures in Differential Geometry SS 2013

Sheet 05

Exercise 23

Show that the restriction of the functor $\mathbf{Man} \to \mathbf{Diff}$, $M \mapsto D_M$ to the category $\mathbf{Man}_{fin}$ of finite-dimensional manifolds is fully faithful.

Exercise 24

Let Y, Y', Z, Z' be diffeological spaces and $f\colon Y' \to Y$, $g\colon Z \to Z'$ be morphisms in $\mathbf{Diff}$. Show that

$$\underline{\mathbf{Diff}}(Y, Z) \to \underline{\mathbf{Diff}}(Y', Z'), \quad \varphi \mapsto g \circ \varphi \circ f$$

is a morphism in $\mathbf{Diff}$.

Exercise 25

Show that the category $\mathbf{Diff}$ of diffeological spaces has arbitrary (small) colimits.

Exercise 26

Show that a morphism $f\colon Y \to Y$ of principal G-bundles ver Z is in local trivialisations

$$\Phi_i' \circ f \circ \Phi_i^{-1}\colon U_i \times X \to U_i \times X, \quad (y, x) \mapsto (y, \xi_i(y, x))$$

always given by $(y, g) \mapsto \xi_i(y) \cdot g$, where $\xi_i\colon U_{ij} \to G$ is smooth. Deduce from this that morphisms of principal bundles are automatically isomorphisms.

Exercise 27

Let $\pi\colon Y \to Z$ be a principal G-bundle. Show that $Y \times_Z Y$ is isomorphic to the trivial principal G-bundle over Y. **Hint:** $(y, y') \in Y \times_Z Y$ gives rise to a unique $g \in G$ with $y' = y \cdot g$.

Exercise 28

Let $\pi\colon Y \to Z$ be a bundle. Show that

$$(U \overset{\circ}{\subseteq} Z) \mapsto \Gamma^\pi(U) := \{\gamma \in C^\infty(U, Y) \mid \pi \circ \gamma = \mathrm{id}_U\}$$

is a sheaf on $\mathbf{Open}_X$. This is also called the *sheaf of sections* of $\pi\colon Y \to Z$.