# Exercises for Algebra II, WS 12/13

## Sheet 8

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### Exercise 35

Let $\mathcal{C}$ be an additive category.

a) Show that if $\mathcal{C}$ has only one object, the $\mathcal{C}$ is actually a ring (how is the corresponding precise statement in terms of an equivalence of categories?).

b) Let $\mathrm{Fun}^{\mathrm{add}}(\mathcal{C}, \mathbf{Ab})$ be the category of additive functors from $\mathcal{C}$ to $\mathbf{Ab}$. Show that this is again an additive category.

c) Show that if $\mathcal{C}$ has only one object and $R$ is the associated ring, then $\mathrm{Fun}^{\mathrm{add}}(\mathcal{C}, \mathbf{Ab})$ is equivalent to **R-Mod**.

### Exercise 36

Let $R$ be a ring and consider the category $\mathrm{Ch}(R)$ of chain complexes of $R$-modules (a morphism $A^\bullet \to B^\bullet$ is a famitly $f^\bullet$ of $R$-module morphisms making the respective diagram commute). Show that the complex $\ker(f^\bullet)$ is a kernel of $f^\bullet$ and that the compex $\mathrm{coker}(f^\bullet)$ is a cokernel of $f^\bullet$.

### Exercise 37

Show that the following are abelian categories (with respect to the usual additive structure):

a) The category $\mathbf{Ab}^{\mathrm{fin}}$ of all finite abelian groups.

b) The category $\mathrm{Ch}(R)$ of chain complexes of $R$-modules.

Show also that in the category **TopAb** of Hausdorff topological abelian groups not each morphism has a cokernel (**Hint:** show first that a morphism with dense image is an epimorphism).

### Exercise 38

Show the **9-Lemma** (or $3 \times 3$-Lemma): Suppose

$$\begin{tikzcd}
A_1 \arrow[r] \arrow[d] & B_1 \arrow[r] \arrow[d] & C_1 \arrow[d] \\
A_2 \arrow[r] \arrow[d] & B_2 \arrow[r] \arrow[d] & C_2 \arrow[d] \\
A_3 \arrow[r] & B_3 \arrow[r] & C_3
\end{tikzcd}$$

is a commuting diagram of morphisms of $R$-modules such that the rows are short exact and the composition of the vertical morphisms is zero. Then the three colums are short exact if and only if two arbitrary of them are so.