Research Seminar on Differential Geometry

Group Analysis and Geometry, University of Hamburg

summer term 2017

The seminar takes place on Mondays at 4 p.m. (c.t.) in the room 142 (Geom) unless noted otherwise.

April 3rd: Wolfgang Globke (Adelaide): **Compact pseudo-Riemannian homogeneous spaces**

Abstract: Let M be a pseudo-Riemannian homogeneous space of finite volume. Then M can be presented as $M = G/H$, where G is a Lie group acting transitively and isometrically on M, and H is a closed subgroup of G. The condition that G acts isometrically and thus preserves a finite measure on M leads to strong algebraic restrictions on G. In the special case where G has no compact semisimple normal subgroups, it turns out that the isotropy subgroup H is a lattice, and that the metric on M comes from a bi-invariant metric on G. This result allows us to recover Zeghib's classification of Lorentzian compact homogeneous spaces, and to move towards a classification for metric index 2. This talk is based on joint works with Oliver Baues and Abdelghani Zeghib.

April 5th: Lana Casselmann (Hamburg): **Localization for K-Contact Manifolds**

Note that the talk will take place at 2 p.m. (c.t.) in room 415 (Geom).

Abstract: The Jeffrey-Kirwan residue formula computes the intersection pairings on a symplectic quotient $N//G$ as the residues of certain meromorphic differential forms associated to the fixed point set M^G, where G is a torus. Key ingredients of the proof are equivariant integration and Atiyah-Bott-Berline-Vergne localization. We extend these techniques to the basic setting on K-contact manifolds and obtain an analogous residue formula. In the special case that $M \to N$ is an equivariant Boothby-Wang fibration, our formulae reduce to the usual ones for the symplectic manifold N.

April 6th: Axel Kleinschmidt (AEI, Potsdam): **Rigid Calabi-Yau manifolds and Picard modular groups**

The talk had to be canceled.

April 10th: Stefan Suhr (Hamburg): **Lyapunov functions for closed cone fields: Conley theory and time functions**

Abstract: Lyapunov functions are functions increasing along orbits of a dynamical system (e.g. a flow of a vector field). It has been known since the work of Conley and Hurley that the common critical set of all such differentiable functions is closely related to the dynamics of the systems. Independently Geroch and Hawkins proved earlier that the existence of a time function for a Lorentzian manifold is strongly related to the "causal structure" of the spacetime. Seen from a elevated point of view both notions express similar ideas with different goals though. The relation was noticed by Fathi/Siconolfi and used to prove existence of time functions for continuous cone fields.

I will introduce the ideas to a theory "à la Conley" for closed cone fields unifying the cases of continuous vector fields with singularities, differential inclusions, Lorentzian metrics and continuous cone fields. I will further discuss the equivalence of global hyperbolicity and the existence of "steep" time functions for closed cone fields, generalizing results of Bernal/Sanchez and Müller/Sanchez. The last result is especially interesting for solutions of the Einstein equations with low regularity.

May 8th: Felix Lubbe (Hamburg): **Mean Curvature Flow of 2-Dimensional Graphs**

Abstract: Let $f : M \to N$ be a smooth map between two Riemannian manifolds M, N, where M is compact and N complete. Each such f defines another map $F : M \to M \times N$, given by $F(x) = (x, f(x))$, and the corresponding graph is given by

$$\Gamma_f := \{(x, f(x)) : x \in M\} = F(M).$$

I will discuss the mean curvature flow of such graphs, i. e. the parabolic system

$$\partial_t F_t(x) = H(x,t), \qquad F_0(x) = (x, f(x)), \quad \forall x \in M,$$

where $H(x,t)$ denotes the mean curvature vector of the graph at $F_t(x)$. Under some assumptions on the sectional curvatures of M and N, and if the initial function f satisfies a length-decreasing (resp. area-decreasing) property, one can show that each $F_t(M)$ is the graph of a map f_t that is again length-decreasing (resp. area-decreasing). Further, for the case of M and N being Riemannian surfaces, I will discuss how decay estimates for the extrinsic curvature quantities can be obtained, which imply long-time existence of the flow.

May 19th: Martin Kell (Tübingen): **On quotient of spaces with Ricci curvature bounded below**

Note that the talk will take place at 10 a.m. (c.t.) in room 531 (Geom).

Abstract: It is well known that a lower bound of the sectional curvature of Riemannian manifold is an Alexandrov space with the same lower sectional curvature

bound. Moreover, the analogous stability property holds for metric foliations and submersions.

In this talk I present the corresponding stability properties for synthetic Ricci curvature lower bounds which are defined via optimal transport theory. More generally, the stability result holds for spaces satisfying the measure contraction property.

Using the structure theory of spaces with Riemannian Ricci curvature bounded below, so-called RCD*(K,N)-spaces, generalizations of Kobayashi's Classification theory of homogeneous Riemannian manifolds follow.

Those results directly apply to Ricci curvature characterization of orbifolds and - via the recent theory of Sturm - also to (super-)Ricci flows.

This is joint work with F.Galaz-Garcia, A.Mondino and G. Sosa.

May 22nd: Jakob Hedicke (Hamburg): **Linear stability of power law inflation models**

Abstract: Given some globally hyperbolic background solution of the Einstein equation $Ric - \frac{1}{2} scal_g g = T$ an important question is, how solutions that are similar to this background solution for some time t_0, behave for later times. Since in general it is difficult to prove results about this, it can be useful to look at the linearised version of this problem.

Using the ADM-decomposition of globally hyperbolic manifolds I will linearise the Einstein equation around a background solution. The solutions of these linearised equations describe "small" perturbations of the background metric, which are for example used in physics to describe gravitational waves. Furthermore I will show how these perturbations can be characterised in the case where the background metric is a power law inflation model on a manifold with closed, Ricci-flat Cauchy-hypersurface.

May 29nd: Áron Szabó (Hamburg): **Stability of compact warped product Einstein manifolds**

Abstract: Perelmans celebrated entropy functional is a quantity which is stationary at Ricci solitons, in particular at Einstein metrics, under the Ricci flow. Considering the second variation of this functional, we obtain a notion of stability for Einstein metrics in terms of the spectrum of a certain operator. There is a related notion of stability which is obtained considering generalized Schwarzschild–Tangherlini black holes. Following the preprint [BHF2016], I will discuss these two concepts of stability of Einstein metrics on compact manifolds, and explain some destabilising criteria.

[BHF2016] W. Batat, S. Hall, T. Murphy, *Destabilising compact warped product Einstein manifolds*, arXiv:1607.05766v1

June 26nd: Klaus Krönke (Hamburg): **Stability of ALE gravitational Instantons under Ricci flow**

Abstract: An ALE gravitational instanton is a four-dimensional hyperkhler manifold that looks like the four-dimensional euclidean space at large distances. As hyperkhler manifolds are Ricci flat, they are in particular stationary points of Hamilton's Ricci flow. We will show that every ALE gravitational instanton (M, g) is stable under Ricci flow in the following sense: Any Ricci flow starting in a suitable small neighbourhood of g in the space of metrics exists for all time and converges modulo diffeomorphism to another ALE gravitational instanton close to g. This is joint work with Alix Deruelle.

July 3rd Stacey Harris (St. Louis): **Is the sky finite or infinite?**

Abstract: "The sky" is future null infinity of spacetime. The simplest model is Minkowski space, with future null infinity being $\mathbb{R}^1 \times S^2$ (a sphere's worth of directions, persisting for a line's worth of time). There is no metric on a null hypersurface to measure finite or infinite extent; but we can still gauge finite vs. infinite if we have a linear (or affine) connection on that hypersurface. What connections exist "naturally" on future null infinity (i.e., the future causal boundary) of a spherically symmetric standard-static spacetime, similar in boundary to Minkowski space? The answer is, there is a family of reasonable connections, parametrized by an arbitrary function from $\mathbb{R}$ to $\mathbb{R}$; simple choices for this function (for example, constant) yield a complete connection on the boundary. If one employs the more common, but less general, conformal boundary construction, the result is the same. Other possible boundaries, such as timelike $\mathbb{R}^1 \times S^2$ at infinity for Anti-deSitter space, timelike $\mathbb{R}^1$ at the singularity for Reissner-Nordström, timelike $\mathbb{R}^1$ for the "boundary" at the origin for Minkowski space, or spacelike $\mathbb{R}^1 \times S^2$ at the singularity for Schwarzschild space all have a much simpler answer: there is a unique connection that fits with the spacetime, and it is complete.