Prof. Dr. J. Struckmeier WS 2009/10

Tutorial on "Advanced Fluid Dynamics"
Deadline: 3rd December 2009

Exercise 8 The Stokes equations given by

$$\begin{aligned} -\Delta u + \nabla p &= f \\ \operatorname{div} u &= 0 \end{aligned}$$

may be *regularized* using a penalty term to get

$$\begin{aligned} -\Delta u^\varepsilon + \nabla p^\varepsilon &= f \\ \varepsilon p^\varepsilon + \operatorname{div} u^\varepsilon &= 0 \end{aligned}$$

where $\varepsilon > 0$.
Formulate the weak form of the regularized Stokes equations with homogeneous Dirichlet data and try to find an estimate for the error $\|u - u^\varepsilon\|_V + \|p - p^\varepsilon\|_W$.

Exercise 9 Consider the example from the lecture, namely the one–dimensional Helmholtz equation on $[0,1]$ given by

$$-u'' + \omega^2 u = f, \quad u(0) = u(1) = 0$$

Compute the stiffness matrix A using the piecewise linear C^0–elements from the lecture.
Build up an FE–method using the function $f(x) = \sin \pi x$ and compare your approximation with the exact solution.