Prof. Dr. J. Struckmeier WS 2009/10

Tutorial on "Advanced Fluid Dynamics"

Deadline: 19th November 2009

Exercise 5 The Hagen–Poiseuille law

$$u(r) = -\frac{Re}{4}\frac{\partial p}{\partial x}(R^2 - r^2)$$

describes the velocity profile of a laminar cylinder–symmetric flow of an incompressible fluid through a pipe with radius R aligned in the x–direction.
Derive this law from the incompressible Navier–Stokes model (neglecting the gravity force) and using the *no–slip* condition $u = 0$ at the interior surface of the pipe. Compute the mass that flows through a cross–section Q of the pipe per second.

Exercise 6 Reformulate the incompressible Navier–Stokes equations

$$\begin{aligned} \operatorname{div} u &= 0 \\ \frac{Du}{Dt} + \nabla p &= \frac{1}{Re}\Delta u + \frac{1}{Fr} f \end{aligned}$$

in terms of the vorticity $\omega = \nabla \times u$ assuming that the force is conservative. How does the vorticity–velocity formulation simplifies in the two–dimensional case?

Exercise 7 For the description of turbulent flows one often uses the so–called *Reynolds–averaged Navier–Stokes equations* (RANS). Here a flow quantity v is expressed as the sum of two terms

$$v = \bar{v} + v'$$

where $\bar{v}$ denotes the mean value and v' the fluctuation around the mean. The mean is usually defined as the time average over the interval $\mathcal{T}$

$$\bar{v} = \frac{1}{\mathcal{T}} \int_{t-\mathcal{T}/2}^{t+\mathcal{T}/2} v(\tau)\, d\tau$$

Show that the RANS model of the equations

$$\begin{aligned} \operatorname{div} u &= 0 \\ \rho\frac{Du}{Dt} &= \operatorname{div} \sigma + \rho f \end{aligned}$$

(with stress tensor σ) is given by

$$\begin{aligned} \operatorname{div} \bar{u} &= 0 \\ \rho\frac{D\bar{u}}{Dt} &= \operatorname{div}(\bar{\sigma} + R) + \rho \bar{f} \end{aligned}$$

where R denotes the (symmetric) Reynolds stress tensor with

$$R_{ij} = \rho\overline{u_i' u_j'}$$