Prof. Dr. J. Struckmeier WS 2009/10

Tutorial on "Advanced Fluid Dynamics"
Deadline: 5th November 2009

Exercise 1 Show for a two–dimensional incompressible flow that

$$\operatorname{rot}((u \cdot \nabla)u) = (u \cdot \nabla) \operatorname{rot} u$$

as well as

$$\operatorname{rot}((u \cdot \nabla)u) = (u \cdot \nabla) \operatorname{rot} u - ((\operatorname{rot} u) \cdot \nabla)u$$

in the three–dimensional case.

Exercise 2 The *viscous part* of the stress tensor for a Newtonian fluid is given by

$$\tau = \lambda \operatorname{div} u \cdot I + 2\mu\gamma$$

with $\lambda = -\frac{2}{3}\mu$ and

$$\gamma = \frac{1}{2}\left(\nabla u + (\nabla u)^T\right)$$

denotes the *rate of strain tensor*.
Verify the relation

$$\operatorname{div} \tau = \frac{\mu}{3} \operatorname{grad} \operatorname{div} u + \mu \Delta u$$

Exercise 3 For a *perfect gas* the equations of state are given by

$$p = \rho R T, \qquad e = e(T)$$

where e denotes the internal energy. Furthermore, the enthalpy h and the entropy s are in general defined by

$$h = e + \frac{p}{\rho}, \qquad T ds = de + p\, d\left(\frac{1}{\rho}\right)$$

Show that the entropy s for a perfect gas can be written as

$$s(T,p) = c_p \ln \frac{T/T_0}{(p/p_0)^{(\kappa-1)/\kappa}}$$

where κ denotes the adiabatic exponent.

Exercise 4 The energy equation for a compressible fluid can be written as

$$\rho \frac{De}{Dt} = -p \operatorname{div} u + \Pi$$

where $\Pi = \Pi(\Psi, \operatorname{grad} T, q)$ includes the dissipation, the heat flux into the system and the internal heat production. Show that the energy equation can be re–written in terms of the enthalpy as

$$\rho \frac{Dh}{Dt} = \frac{Dp}{Dt} + \Pi$$