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Simple Lie Algebras over Fields of Positive Characteristic

I. Structure Theory

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The problem of classifying the finite-dimensional simple Lie algebras over fields of characteristic $p > 0$ is a long-standing one. Work on this question during the last 35 years has been directed by the Kostrikin–Shafarevich Conjecture of 1966, which states that *over an algebraically closed field of characteristic $p > 5$ a finite-dimensional restricted simple Lie algebra is classical or of Cartan type*. This conjecture was proved for $p > 7$ by Block and Wilson in 1988. The generalization of the Kostrikin–Shafarevich Conjecture for the general case of not necessarily restricted Lie algebras and $p > 7$ was announced in 1991 by Strade and Wilson and eventually proved by Strade in 1998. The final Block–Wilson–Strade–Premet Classification Theorem is a landmark result of modern mathematics and can be formulated as follows: *Every finite-dimensional simple Lie algebra over an algebraically closed field of characteristic $p > 3$ is of classical, Cartan, or Melikian type*.

In the two-volume book, the author is assembling the proof of the Classification Theorem with explanations and references. The goal is a state-of-the-art account on the structure and classification theory of Lie algebras over fields of positive characteristic leading to the forefront of current research in this field.

This first volume is devoted to preparing the ground for the classification work to be performed in the second volume. The concise presentation of the general theory underlying the subject matter and the presentation of classification results on a subclass of the simple Lie algebras for all odd primes will make this volume an invaluable source and reference for all research mathematicians and advanced graduate students in algebra.

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