EXERCISES, LIE ALGEBRAS, UNIVERSITY OF HAMBURG, SUMMER SEMESTER 2016

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Sheet 4

Exercise 1. Let $\mathfrak{g}$ be a Lie algebra and let $x \in \mathfrak{g}$. Prove that the vector subspace of $\mathfrak{g}$ spanned by the eigenvectors of $\operatorname{ad} x$ is a Lie subalgebra.

Exercise 2. Show that $\operatorname{ad} \mathfrak{g}$ is an ideal in $\operatorname{Der}(\mathfrak{g})$.

Exercise 3. Prove that a Lie algebra is solvable (resp. nilpotent) if and only if $\operatorname{ad} \mathfrak{g}$ is solvable (resp. nilpotent).

Exercise 4. Let $\mathfrak{g}$ be a finite dimensional Lie algebra and let $I \subset \mathfrak{g}$ be an ideal of $\mathfrak{g}$ such that the quotient algebra $\mathfrak{g}/I$ is nilpotent and such that the restriction of $\operatorname{ad} x$ to I is nilpotent for all $x \in \mathfrak{g}$. Show that $\mathfrak{g}$ is a nilpotent Lie algebra.

Exercise 5. Prove that $\mathfrak{g} = \mathfrak{sl}(3, K)$ is simple if $\operatorname{char}(K) \neq 3$.

Hint: Recall that $\mathfrak{sl}(3, K)$ has as basis the following matrices: $E_{i,j}$ for $1 \leq i, j \leq 3$ and $i \neq j$, and the matrices $H_1 = E_{1,1} - E_{2,2}$, $H_2 = E_{2,2} - E_{3,3}$. Assume $0 \neq I$ is an ideal in $\mathfrak{g}$ and consider the action of $\operatorname{ad} H_1$ and $\operatorname{ad} H_2$ on it. You may use without proof that $\operatorname{ad} H_1$ and $\operatorname{ad} H_2$ are diagonalisable endomorphisms of the eight-dimensional vector space $\mathfrak{g}$.