EXERCISES, COMPLEX GEOMETRY, UNIVERSITY OF HAMBURG, WINTER SEMESTER 2015/2016

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Sheet 9

Exercise 1. (compare [2, Ex. 1.2.2]**)** Show that the decomposition $\bigwedge^k V^* = \oplus_{i\geq 0} L^i P^{k-2i}$ is orthogonal with respect to the Hodge-Riemann pairing.

Exercise 2. (compare [2, Ex. 1.2.10]**)** Let $x_1, y_1 = I(x_1), \ldots, x_n, y_n = I(x_n)$ be an orthonormal basis of a euclidian vector space V which is endowed with a compatible almost complex structure I. Let α be a two form. Show that $\Lambda(\alpha) = \sum_i \alpha(x_i, y_i)$.

Exercise 3. [2, Ex. 2.6.1] Show that every complex manifold admits a hermitian structure.

References

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