EXERCISES, COMPLEX GEOMETRY, UNIVERSITY OF HAMBURG, WINTER SEMESTER 2015/2016

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Sheet 3

Exercise 1. [1, Ex. 2.1.11] Let V be a complex vector space of dimension n. Show that the map which sends a k-dimensional subvector space $W \subset V$ to $\Lambda^k W \subset \Lambda^k V$ is a holomorphic embedding of the Grassmannian $\mathrm{Gr}_k(V)$ into the projective space $\mathbb{P}(\Lambda^k V)$. It is called the *Plücker embedding*. In particular, Grassmannian manifolds are indeed projective as claimed in the lecture.

Exercise 2. Let $\mathcal{F}$ be a sheaf on a topological space X, let $U \subset X$ be an open set and $s \in \mathcal{F}(U)$. The *support* of s, denoted by $\mathrm{supp}(s)$ is defined as $\{p \in X \mid s_p \neq 0\}$, where s_p is the image of s in the stalk $\mathcal{F}_p$. Show that the support of s is a closed subset of X.

Exercise 3. Let X be a topological space, $W \subset X$ be a subset, $p \in X$ be a point and A be an abelian group.

(1) The *closure* of W, denoted by $\overline{W}$, is the smallest closed subset of X containing W, so $\overline{W} = \cap_{W \subset Z} Z$, where the interesection runs over closed subsets containing W. The *boundary* of W, denoted by ∂W is $\{x \in X \mid U \cap A \neq \emptyset \ \forall \text{ open } U \text{ with } x \in U\}$. Show that $\overline{W} = W \cup \partial W$.

(2) Define a presheaf $i_p(A)$ as follows: $i_p(U) = A$ if $p \in U$ and 0 otherwise. Show that $i_p(A)$ is a sheaf and that $(i_p(A))_q = A$ for any q in $\overline{\{p\}}$, while $(i_p(A))_q = 0$ otherwise. The sheaf $i_p(A)$ is called the *skyscraper sheaf*.

Exercise 4. Let $\varphi \colon \mathcal{F} \longrightarrow \mathcal{G}$ be a morphism of sheaves. Show that $\ker(\varphi)$ is a sheaf.

References

[1] D. Huybrechts, *Complex geometry: An introduction*, Springer, Berlin (2005).
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[3] C. Schnell, *Complex manifolds*, available at `http://www.math.stonybrook.edu/~cschnell/`.
[4] C. Voisin, *Hodge theory and complex algebraic geometry*, Cambridge University Press, Cambridge (2002).