

EXERCISES, COMPLEX GEOMETRY, UNIVERSITY OF HAMBURG,  
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SHEET 1

**Exercise 1.** [1, Ex. 1.1.1] Show that every holomorphic map  $f: \mathbb{C} \rightarrow \mathbb{H} = \{z \mid \text{im}(z) > 0\}$  is constant.

**Exercise 2.** [1, Ex. 1.1.2] Show that the real and imaginary part of a holomorphic function  $f = u + iv$  are harmonic.

**Exercise 3.** Show that the image of any holomorphic non-constant map  $f: \mathbb{C} \rightarrow \mathbb{C}$  is dense (recall that  $f$  has dense image if for any  $z \in \mathbb{C}$  and all  $\epsilon > 0$ ,  $B_\epsilon(z) \cap \text{im}(f) \neq \emptyset$ ).

REFERENCES

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