EXERCISES, COMMUTATIVE ALGEBRA, UNIVERSITY OF HAMBURG, WINTER SEMESTER 2014/2015

P. SOSNA

Sheet 3

Exercise 1. [1, Ex. 2.2 & 2.3] Let A be a ring, M be an A-module and I be an ideal in A. Show that $A/I \otimes_A M \simeq M/IM$. Use this to prove the following statement. If M and N are finitely generated modules over a local ring A, then $M \otimes_A N \simeq 0$ implies that either $M \simeq 0$ or $N \simeq 0$.

Exercise 2. [1, Ex. 2.4 & 2.5] Prove that a direct sum of modules is flat if and only if every summand is flat. Use this to show that for any ring A the module $A[X]$ is flat.

Exercise 3. [1, Ex. 2.25] Let $0 \longrightarrow M' \longrightarrow M \longrightarrow M'' \longrightarrow 0$ be an exact sequence of A-modules with M'' flat. Prove that M is flat if and only if M' is flat.

Exercise 4. [1, Ex. 3.1] Let A be a ring, S a multiplicatively closed subset of A and M be a finitely generated A-module. Show that if $S^{-1}M = 0$, then there exists an element $s \in S$ such that $sM = 0$.

References

[1] M. F. Atiyah and I. G. MacDonald, *Introduction to commutative algebra*, Addison-Wesley Publishing Co., Reading Mass.-London-Don Mills, 1969.
[2] P. L. Clark, *Commutative algebra*, http://www.math.uga.edu/ pete/integral.pdf.
[3] Q. Liu, *Algebraic geometry and arithmetic curves*, Oxford University Press, Oxford, 2002.
[4] H. Matsumura, *Commutative ring theory*, 2nd ed., Cambridge Studies in Advanced Mathematics, Cambridge University Press, Cambridge, 1989.