Complex Geometry
WS 16/17

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Exercises 9

1. Let X be a complex manifold, $\mathbb{Z}_X$ the constant sheaf on X with fibres $\mathbb{Z}$ and $\mathcal{O}_X^\times$ the multiplicative sheaf of holomorphic functions without zeroes. Check in detail exactness of the exponential sequence

$$0 \longrightarrow \mathbb{Z}_X \longrightarrow \mathcal{O}_X \xrightarrow{\exp} \mathcal{O}_X^\times \longrightarrow 0,$$

with $\exp(f) = e^{2\pi\sqrt{-1}f}$ for a holomorphic function f on an open subset $U \subset X$.

2. Let $\varphi : \mathcal{F} \to \mathcal{G}$ be a homomorphism of abelian sheaves. Show that there is a canonical isomorphism

$$\operatorname{im}\varphi \simeq \mathcal{F}/\ker\varphi.$$

Conclude that if φ is a monomorphism ($\ker\varphi = 0$) then $\mathcal{F}$ can be identified with a subsheaf of $\mathcal{G}$.

3. Let $\mathfrak{U} = \{U_0, U_1\}$ be the standard covering of $\mathbb{P}^1$ and let $\mathcal{I} \subset \mathcal{O}_{\mathbb{P}^1}$ be the ideal sheaf generated by the holomorphic function $(z_1/z_0)^2$ on U_0 and otherwise equal to $\mathcal{O}_{\mathbb{P}^1}$. Compute the Čech cohomology group $H^1(\mathfrak{U}, \mathcal{I})$ (answer: $\mathbb{C}$).