Complex Geometry
WS 16/17

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Exercises 6

1. Let $\lambda \in \mathbb{C}$, $0 < |\lambda| < 1$. Consider the $\mathbb{Z}$-action on $\mathbb{C} \setminus \{0\}$ defined by $k.z = \lambda^k z$ for $k \in \mathbb{Z}$. Show: The quotient $(\mathbb{C} \setminus \{0\})/\mathbb{Z}$ (a one-dimensional Hopf-manifold) is isomorph to an elliptic curve $\mathbb{C}/(\mathbb{Z} + \mathbb{Z}\tau)$. Determine also $\lambda \in \mathbb{C}^*$ as a function of $\tau \in \mathbb{H}$.

2. Find a holomorphic map $f : X \to Y$ between connected complex manifolds such that any elliptic curve $\mathbb{C}/(\mathbb{Z} + \mathbb{Z}\tau)$, $\tau \in \mathbb{H}$, is isomorphic to a fibre of f.

3. Let $X = V(x^3 + y^3 + z^3 + w^3) \subset \mathbb{P}^3$ be the Fermat cubic. Find 12 lines $L_1, \ldots, L_6$, $L'_1, \ldots, L'_6$ on X with the following properties:

(i) $\forall i \neq j : \quad L_i \cap L_j = \emptyset,\ L'_i \cap L'_j = \emptyset,\ L_i \cap L'_j \neq \emptyset.$
(ii) $\forall i : \quad L_i \cap L'_i = \emptyset.$

(Such a configuration of lines is called *Schläfli double six.*)