Complex Geometry
WS 16/17

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Exercises 5

1. Find an irreducible analytic set $X \subset \mathbb{C}^3$ such that the projection

$$\pi : X \longrightarrow \mathbb{C}, \quad (z_1, z_2, z_3) \longmapsto z_3$$

has the following property: All fibres $\pi^{-1}(z)$ for $z \neq 0$ are reducible, but $\pi^{-1}(0)$ is irreducible. (Hint: see Sheet 4)

2. Let $f = z_2^2 - z_1^3$ and $X \subset \mathbb{P}^2$ the closure of $Z(f) \subset U_0$ in $\mathbb{P}^2$. For $i = 1, 2$ and $U_i = \big\{ [z_0, z_1, z_2] \in \mathbb{P}^2 \,\big|\, z_i \neq 0 \big\} \simeq \mathbb{C}^2$ find $f_i \in \mathcal{O}(U_i)$ with $X \cap U_i = Z(f_i)$.

3. (a) Show that $\mathbb{P}^n$ is compact.

(b) Show that any non-discrete complex submanifold $X \subset \mathbb{C}^n$ is non-compact.

4. Let X be a connected compact complex manifold and $Y \subset \mathbb{C}^n$ a complex submanifold. Show that any holomorphic map $f : X \to Y$ is constant.