Complex Geometry
WS 16/17

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Exercises 2

1. Discuss uniqueness of the decomposition in the Weierstrass preparation theorem.

2. Decompose $f(z_1, z_2) = z_1^3 z_2 + z_1 z_2 + z_1^2 z_2^2 + z_2^2 + z_1 z_2^3$ according to the Weierstrass preparation theorem.

3. Let $U \subset \mathbb{C}^m$ be open, $P \in U$ and $f : U \to \mathbb{C}^n$ holomorphic with $J(f)(P)$ of maximal rank.

a) (*Holomorphic submersion*) Let $m \geq n$, that is, $J(f)(P)$ surjective.

Show: There exists a biholomorphic map $\Phi : V \longrightarrow U'$, $U \subset U$ an open neighbourhood of P, and $V \subset \mathbb{C}^m$, such that

$$(f \circ \Phi)(z_1, \ldots, z_m) = (z_1, \ldots, z_n).$$

(Interpretation: If $J(f)$ is surjective, in appropriate holomorphic coordinates, f is a linear projection.)

b) (*Holomorphic immersions*) Let $m \leq n$, that is, $J(f)(P)$ injective.

Show: There exists a biholomorphic map $\Psi : V \longrightarrow V'$, $V \subset \mathbb{C}^n$ an open neighbourhood of $f(P)$, and $V' \subset \mathbb{C}^n$ with

$$(\Psi \circ f)(z_1, \ldots, z_m) = (z_1, \ldots, z_m, 0, \ldots, 0).$$

(Interpretation: If $J(f)$ is injective, in appropriate holomorphic coordinates f is the inclsuion of a linear subspace.)

4. Let $U \subset \mathbb{C}^n$ be open and connected and $f \in \mathcal{O}(U)$. Show that $U \setminus Z(f) \subset U$ is connected and dense. [Hu, Ex. 1.1.8]