Complex Geometry Prof. Dr. Bernd Siebert
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Exercises 10

1. Let L be the tautological line bundle on $\mathbb{P}^1$ with fibre through a point $[z_0, z_1] \in \mathbb{P}^1$ the line $\mathbb{C} \cdot (z_0, z_1)$. For a projective line $G \subset \mathbb{P}^n$, $n \geq 1$, describe $T_{\mathbb{P}^n}|_G$ by a cocycle and then show $\det \big(T_{\mathbb{P}^n}|_G\big)^* \simeq L^{\otimes(n+1)}$.

2. Let $\mathcal{L}$ be the sheaf of holomorphic sections of the tautological line bundle L on $\mathbb{P}^1$ (see Ex.10.1). Show that $\mathcal{L}$ is isomorphic to the ideal sheaf $\mathcal{I}$ of a point $p \in \mathbb{P}^1$, say of $p = [0, 1]$.

3. Let L be the tautological line bundle on $\mathbb{P}^n$ and $S_0 \subset L$ the image of the zero section. Define a holomorphic map

$$\pi : L \to \mathbb{C}^{n+1}$$

with $\pi\big(S_0\big) = 0$ and such that π restricts to an isomorphism $L \setminus S_0 \to \mathbb{C}^{n+1} \setminus \{0\}$. (Hint: π is isomorphic to the *blowing up* of $\mathbb{C}^{n+1}$ at the point 0.)

4. Show that the holomorphic tangent bundle T_X of a complex torus $X = \mathbb{C}^n/\Lambda$ is trivial.

5. Let L be a holomorphic line bundle on a compact connected complex manifold X. Show that L is trivial if and only if L and L^* admit non-trivial global sections. (Hint: Use the sections to construct a non-trivial section of $L \otimes L^* \simeq \mathcal{O}_X$).