Arbeitsgemeinschaft mit aktuellem Thema:
Algebraic structures in conformal field theories
Mathematisches Forschungsinstitut Oberwolfach
01.04.-07.04.2007

Organizers:

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Introduction:

Two-dimensional conformal field theory plays a fundamental role in the theory of two-dimensional critical systems of classical statistical mechanics, in quasi one-dimensional condensed matter physics and in string theory. The study of defects in systems of condensed matter physics, of percolation probabilities and of (open) string perturbation theory in the background of certain solitonic solutions of string theory, the so-called D-branes, forces one to analyze conformal field theories on surfaces that may have boundaries and / or can be non-orientable. This study has recently led to much new insight in the mathematical structure of conformal field theory.

Many mathematical disciplines have contributed to a better understand of conformal field theory and received stimulating input from questions arising in conformal field theories. There are two major approaches to chiral conformal field theory: one that is based on operator algebras and one based on vertex algebras. Both structures lead to representation categories that are tensor categories and, in the case of rational chiral conformal field theories, more specifically modular tensor categories. They also encode the monodromy representations of the vector bundles of conformal blocks for rational vertex algebras, objects that are of interest for algebraic geometry. Moreover, modular tensor categories are a crucial ingredient in the construction of three-dimensional topological field theories.

While chiral conformal field theories have certain physical applications in the description of quantum Hall systems, full local conformal field theories are relevant for the physical applications referred to in the first paragraph. Recently, it has been understood that the construction of a full local conformal field theory is best described using the structure of a module category over the tensor category that describes the chiral data.

In this Arbeitsgemeinschaft, we will explain this mathematical notion, related concepts and some applications. The category theoretic framework allows to emphasize those aspects that are common to all approaches to chiral conformal field theory.

Talks:

(as of February 12, 2007)
The talks are marked by the organizer in charge with whom details of the talk should be primarily discussed.

Introductory talk

1. **Tensor categories and module categories** (Rolf Dyre Svegstrup) [CS]
   Introduce the notions of a tensor category and of a module category over a tensor category. Introduce the notions of a braiding and a duality for a tensor category.

   References: [K, O]

Examples

2. **Tensor categories from factors** (Arthur Bartels) [YK]
   Explain how tensor categories arise from nets of factors in the operator algebraic framework. This is an adaption of the classical Doplicher-Haag-Roberts theory to CFT.

   References: [FRS, KLM]

3. **Tensor categories for loop groups** (Ulrich Bunke) [CS]
   Explain important facts about the representation categories for WZW models . The emphasis should be: description of simple objects, structure of K_0, including the Verlinde formula, and possibly the extended Racah-Speiser algorithm [F, Section 5.5].

   References: [B, Part II, III]

4. **Frobenius algebras and Q-systems** (Pinhas Grossman) [YK]
   Explain the notion of a Q -ystem is and of a Frobenius algebra in a tensor category is and their relation.

   References: [Lon, LoRo]

Fusion categories

5. **The double of a finite group** (Jürgen Müller) [VO]
Present the representation category of the double of a finite group as an example for (modular) tensor categories. Explain the notion of a modular tensor category. Classification of module categories over group theoretical categories (like double of a finite group) [O4]

   REFERENCES: [BK, Chapter 3.2], [O4]

6. **Orbifolds** (Oliver Gray) [CS]
Present the theory of orbifolds in the categorical framework [Kir] and relate it to its operator algebraic realization [Mue]

   REFERENCES: [Kir], [Mue]

7. **Classification of module categories** (Ho Hai Phung) [VO]
ADE classification for fusion categories associated with SL(2) [EO]; a brief discussion of [MOV] can be included.

8. **Classification of fusion categories** (Michael Cuntz) [VO]
Fusion categories with few objects [O2]; first review the known results and then give some details for one example of rank 2. (Related material includes [O3], categories of specific Frobenius-Perron dimension like pq, p^n, where p, q are primes [EGO]; classification of categories with prescribed K_0 [TY, TW].)

   REFERENCES: [O2, O3, EGO, TY, TW]

9. **Exceptional tensor categories in subfactor theory** (Emily Peters) [YK]
In subfactor theory, some exceptional tensor categories have been constructed by Asaeda, Haagerup and Izumi in [AH] and Section 7 of [I]. These examples are not known in other approaches to CFT and they produce non-trivial examples of topological field theories. Explain these examples and related concrete computations.

   REFERENCES: [AH, I]

Topological field theory

10. **Reshetikhin-Turaev construction** (Rafal Suszek) [CS]
First explain the idea to describe quantum field theory as a functor from a category of cobordisms to an algebraic category [A]. Then give an overview of the Reshetikhin-Turaev construction of a TFT based on a modular tensor category. Possible references are the books [BK, K].

REFERENCES: [A, BK, K]

11. **Three-dimensional TFT from a subfactor** (Chris Schommer-Pries) [YK]
Explain the construction of a (non-braided) tensor category and a three-dimensional TFT from a subfactor.

    REFERENCES: [EK, Chapters 9 and 12]

<u>Conformal blocks</u>

12. **Definition for WZW models** (Vincent Graziano) [VO]
Explain the definition of conformal blocks, for simplicity only for WZW theories

    REFERENCES: [B, Loo]

13. **Knizhnik-Zamolodchikov connection** (Marc Nieper-Wißkirchen) [VO]
Explain the construction of the Knizhnik-Zamolodchikov connection for WZW conformal blocks

    REFERENCES: [BK, Chapter 7.4] [M, Section 2]

<u>CFT correlators</u>

14. **The TFT approach to CFT correlation functions** (Christian Lehn) [CS]
Explain the axioms for correlation functions of a conformal field theory [FjFRS, Section 3] and how the TFT approach solves these constraints [RFS06].

    REFERENCES: [FjFRS, RFS06]

15. **Symmetries and dualities in conformal field theory** (Pasquale Zito) [CS]
Explain how symmetries and dualities of a conformal field theory are encoded in the fusion rules of a bimodule category that describes topological defect lines.

    REFERENCES: [FFRS04, FFRS06]

16. **α-induction** (Marta Asaeda) [YK]
Explain α induction in the categorical framework (section 5 of [O]). Review the status of the 5 claims in [O]:
claim 1 [FRS02]
claim 2 [BEK, Theorem 5.10], [FFRS06, Proposition 4.7, Remark 4.8]
claim 3 [FRS07]
claim 4 [FRS02]
claim 5 [FFRS03]

    REFERENCES: [O]

17. **Boundary conditions in the operator algebraic approach** (Dorothea Bahns) [YK] A review of the algebraic structures arising for conformal field theories on a half-space of Minkowski space-time in the operator algebraic approach to CFT.

    REFERENCES: [LoRe]

The following bibliography contains many hyperlinks; if you view the pdf-version of this file with acroread, you can obtain most papers by clicking on the marked links. The pdf file is available at `http://www.math.uni-hamburg.de/home/schweigert/oberwolfach/program.pdf`

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Participation:

The idea of the Arbeitsgemeinschaft is to learn by giving one of the lectures in the program. If you intend to participate, please send your full name, your e-mail address and your full postal address to

`schweigert@math.uni-hamburg.de`

by February 6, 2007 at the latest. Late applications are considered as long as places are available.

You should also indicate which talk you are willing to give:
First choice: talk no. ...
Second choice: talk no. ...
Third choice: talk no. ...

You will be informed shortly after the deadline if your participation is possible and whether you have been chosen to give one of the lectures.

The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. For information about the institute, see
`http://www.mfo.de/`
The institute offers accommodation and boarding free of charge to the participants. Travel expenses cannot be covered.

Note that the Friday of the week April 1-7 is Good Friday; although this is a public Holiday in Germany, the Institute will operate as usual. We plan to finish the program on Friday early afternoon. Further information will be given to the participants after the deadline.