The Globular Extended QFT of the String propagating on the Classifying Space of a strict 2-Group

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Abstract

The general framework of globular extended QFT is applied to the string propagating on the classifying space of a strict 2-group.

Contents

1 Introduction

This is supposed to be a nontrivial but simple example of a general idea that goes as follows.

There is a mystery that demands to be understood:

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Mystery 1 *The theory of gerbes with connection in terms of local data exhibits a lot of structural resemblance to state sum models of 2-dimensional quantum field theory.*

Why is that?

Does this point to a deeper pattern that we might want to understand?

1.1 Quantization of parallel n-transport

After a little bit of reflection, I think the pattern is

a) n-Bundles with connection are naturally conceived in terms of parallel transport n-functors.

b) Coupling these n-connections to an n-particle amounts to transgressing these n-functors to a suitable configuration space.

c) Quantizing these charged n-particles amounts to pushing the transgressed n-functors forward to a point.

From this point of view, evolution in the quantum field theory of the charged n-particle is an n-functor that is inherently obtained from the parallel transport n-functor that expresses the background field that the particle propagates in.

Both, the original parallel transport n-functor as well as the resulting quantum propagation n-functor may be locally trivialized. For the former this yields the local description of gerbe holonomy. For the latter this yields the state sum description of QFT.

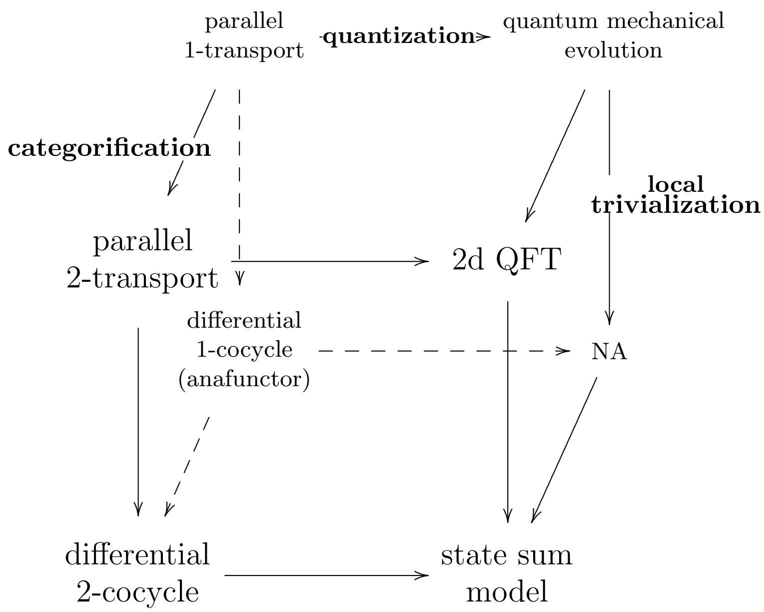


Figure 1: **Quantization, categorification and local trivialization**.

	classical data		quantum theory
	background field	**n-particle**	
name of n-functor	parallel transport	action	quantum propagation
image of n-functor	monodromy	classical phases	quantum amplitudes
	with values in $\mathrm{phas} = n\mathrm{Vect}$		
domain	on target space tar	on configuration space $\mathrm{conf} \subset [\mathrm{par}, \mathrm{tar}]$	on parameter space par
in symbols	$\mathrm{tra} : \mathrm{tar} \to \mathrm{phas}$	$\mathrm{tra}_* : \mathrm{conf} \to [\mathrm{par}, \mathrm{phas}]$	$q(\mathrm{tra}) : \mathrm{par} \to \mathrm{phas}$
operation in physics terms	coupling $\curvearrowright$	quantization $\curvearrowright$	
correspondence	$\mathrm{tar} \xleftarrow{\mathrm{ev}} \mathrm{conf} \times \mathrm{par} \xrightarrow{p} \mathrm{par}$		
operation in symbols	$[\mathrm{tar}, \mathrm{phas}] \xrightarrow{\mathrm{ev}^*} [\mathrm{conf} \times \mathrm{par}, \mathrm{phas}] \xrightarrow{\bar{p}^*} [\mathrm{par}, \mathrm{phas}]$ coupling, quantization $\mathrm{tar} \longmapsto \mathrm{ev}^*\mathrm{tar} \longmapsto \bar{p}^*\mathrm{ev}^*\mathrm{tar}$		
elements	flat sections $e : 1 \to \mathrm{tra}$ in $\Gamma(\mathrm{tra}) = \mathrm{Hom}(1, \mathrm{tra})$	states $\psi : 1_\bullet \to q(\mathrm{tra})$ in $\mathrm{Hom}(1_*, \mathrm{tra}_*) \simeq \mathrm{Hom}(1_\bullet, q(\mathrm{tra}))$	
pairing of elements	holonomy		correlator

Table 1: **The charged n-particle and its quantization**. The process begins with a parallel transport n-functor tra for an n-bundle with connection, modelling a physical background field. It continues by specifying certain maps into the domain of the parallel transport and transgressing tra to the configuration space of all these maps. This models the coupling of the background field to a charged n-particle (a point particle, a string, a membrane, etc.). Finally, the transgressed n-functor may be pushed forward to a point. This yields the quantum theory of the charged n-particle coupled to the given background field.

1.2 Example: string on BG_2

By the *string on the classifying space of a strict 2-group* we here want to understand the following example of a charged n-particle.

Fix a strict 2-group G_2 and set

- $n = 2$
- $\mathrm{par} = \Sigma(\mathbb{Z})$
- $\mathrm{tar} = \Sigma(G_2)\,, \qquad G_2$ a strict 2-group
- $\mathrm{phas} = \mathrm{Bim}$
- $\mathrm{tra} = 1 \in [\Sigma(G_2), \mathrm{Bim}]$

The fact that $n = 2$ means that we are one categorification step above the ordinary theory of point particles coupled to ordinary vector bundles.

The fact that $\mathrm{par} = \Sigma(\mathbb{Z})$, which is the additive group of integers regarded as a category with a single object, means that we are considering 2-particles that look like strings $\bullet \curvearrowright \bullet$ stretching from $\bullet$ to $\bullet$.

The fact that $\mathrm{tar} = \Sigma(G_2)$, which is G_2 regarded as a 2-groupoid with a single object, means that these strings propagate on the classifying space BG_2 of G_2, since the realization $|\Sigma(G_2)|$ of the nerve of G_2 is

$$|\Sigma(G_2)| = BG_2\,.$$

The fact that $\mathrm{phas} = \mathrm{Bim}$ means that everything takes values in those well-behaved 2-vector spaces that are in the image of the canonical inclusion

$$\mathrm{Bim} \hookrightarrow {}_{\mathrm{Vect}}\mathrm{Mod} := 2\mathrm{Vect}\,.$$

We assume that we are working over complex vector spaces.

The fact that $\mathrm{tra} = 1$ means that we consider a trivial rank one 2-vector bundle with trivial connection on target space.

Clearly, in particular this last condition can be replaced by something more interesting.

Given this data, we can determine its quantization:

Proposition 1 *The quantum transport*

$$q(\mathrm{tra}) : \Sigma(\mathbb{Z}) \to \mathrm{Bim}$$

of the above system is

$$q(\mathrm{tra}) : (\bullet \longrightarrow \bullet) \mapsto \mathbb{C}[\Lambda G_2] \xrightarrow{\mathrm{Id}} \mathbb{C}[\Lambda G_2]\ .$$

Here

$$\Lambda G_2 = [\Sigma(\mathbb{Z}), \Sigma(G_2)]/\sim$$

is the groupoid obtained by identifying isomorphic 1-morphisms in configuration space, and

$$\mathbb{C}[\Lambda G_2]$$

is the associated groupoid algebra, over the complex numbers.

This implies that a **state** ψ of the system

$$\psi : 1 \to q(\mathrm{tra})$$

is

- a representation $\psi(\bullet)$ of ΛG_2 over the point $\bullet$
- an endomorphism $\psi(\bullet) \xrightarrow{\psi(\bullet \to \bullet)} \psi(\bullet)$ of this representation over the string $\bullet \to \bullet$.

In string theory terminology, we would call $\psi(\bullet)$ a **D-brane**.

Of special interest is the case where G is a compact, simple and simply connected Lie group, and where

$$G_2 = \mathrm{String}_k(G)$$

is the string 2-group of G, which comes from the level $k \in H^3(G, \mathbb{Z})$.

Then, as discussed elsewhere,

- The state $\psi(\bullet)$ is an AdG-equivariant gerbe module of the gerbe at level k on G.

As also discussed elsewhere, interaction of strings corresponds to the fusion product on these gerbe modules.

2 The quantum theory of the charged n-particle

2.1 The definition

Definition 1 *A* **charged n-particle**

$$\left(\mathrm{par} \xrightarrow{\gamma \in \mathrm{conf}} \mathrm{tar} \xrightarrow{\mathrm{tra}} \mathrm{phas} \right)$$

is

- *an* $(n-1)$*-category* par*, called* **parameter space** *and thought of as modelling the shape and internal structure of the n-particle*
- *an n-category,* tar*, called* **target space** *and thought of as modelling the space that the n-particle propagates in*
- *an n-category* $\mathrm{phas} = n\mathrm{Vect}$*, being the n-category of some notion of n-vector spaces*
- *an n-functor* $\mathrm{tra} : \mathrm{tar} \to \mathrm{phas}$*, thought of as encoding the parallel* **transport** *in an n-bundle with connection on target space*
- *a choice of sub-n-category* $\mathrm{conf} \subset [\mathrm{par}, \mathrm{tar}]$*, thought of as encoding the* **configuration space** *of the n-particle.*

Given a charged n-particle, we obtain the diagram

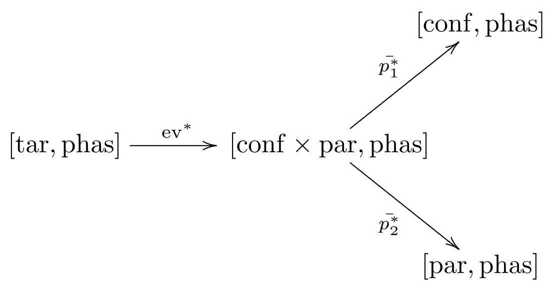
$$\begin{array}{ccccc} & & & & \mathrm{conf} \\ & & & \nearrow^{p_1} & \\ \mathrm{tar} & \xleftarrow{\mathrm{ev}} & \mathrm{conf} \times \mathrm{par} & & \\ & & & \searrow_{p_2} & \\ & & & & \mathrm{par} \end{array} ,$$

where the arrow on the left is the restriction of the canonical evaluation map $\mathrm{ev} : [\mathrm{par}, \mathrm{tar}] \times \mathrm{par} \to \mathrm{tar}$ along the inclusion $\mathrm{conf} \hookrightarrow [\mathrm{par}, \mathrm{tar}]$, and where p_1 and p_2 are the obvious projection on the first and the second factor, respectively.

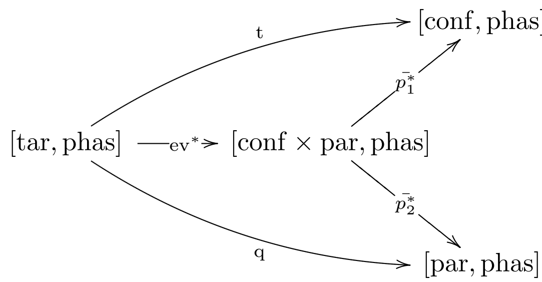
There is a corresponding diagram of pullbacks

$$\begin{array}{ccccc} & & & & [\mathrm{conf}, \mathrm{phas}] \\ & & & \swarrow^{p_1^*} & \\ [\mathrm{tar}, \mathrm{phas}] & \xrightarrow{\mathrm{ev}^*} & [\mathrm{conf} \times \mathrm{par}, \mathrm{phas}] & & \\ & & & \nwarrow_{p_2^*} & \\ & & & & [\mathrm{par}, \mathrm{phas}] \end{array} .$$

If the morphisms on the right have adjoints, $\bar{p}_1^*$ and $\bar{p}_2^*$, respectively, we get



The composition of morphisms along the above route is **transgression**, whereas the composition along the lower route is **quantization**.



Definition 2 *Given a charged n-particle*

$$\left(\mathrm{par} \xrightarrow{\gamma \in \mathrm{conf}} \mathrm{tar} \xrightarrow{\mathrm{tra}} \mathrm{phas} \right) ,$$

its (extended, globular) **quantum theory** *is the image*

$$q(\mathrm{tra}) : \mathrm{par} \to \mathrm{phas}$$

of tra *under this quantization map.*

Remark. It is *extended* because it is an n-functor.

It is *globular* because we think of the globular morphisms in the domain par directly as the extended cobordisms on which the QFT is defined. This means in particular that every n-cobordisms in par has the topology of an n-disk.

The value of our QFT on topologically nontrivial cobordisms will be taken to be its value on any globular cutting of that cobordisms followed by a suitable trace operation.

Caveat. Without further qualification, this definition captures only what would be called the *kinematics* of the quantum theory.

A charged n-particle...	$\left(\mathrm{par} \xrightarrow{\gamma\in\mathrm{conf}} \mathrm{tar} \xrightarrow{\mathrm{tra}} \mathrm{phas} \right)$
... comes with a configuration space of maps from its parameter space into its target space...	$\mathrm{tar} \xleftarrow{\mathrm{ev}} \mathrm{conf}\times\mathrm{par}$, $\quad \mathrm{conf}\times\mathrm{par} \xrightarrow{p_1} \mathrm{conf}$, $\quad \mathrm{conf}\times\mathrm{par} \xrightarrow{p_2} \mathrm{par}$
... and a coupling to a transport functor on target space...	$[\mathrm{tar},\mathrm{phas}] \xrightarrow{\mathrm{ev}^*} [\mathrm{conf}\times\mathrm{par},\mathrm{phas}]$, $\quad [\mathrm{conf},\mathrm{phas}] \xrightarrow{p_1^*} [\mathrm{conf}\times\mathrm{par},\mathrm{phas}]$, $\quad [\mathrm{par},\mathrm{phas}] \xrightarrow{p_2^*} [\mathrm{conf}\times\mathrm{par},\mathrm{phas}]$
...which induces transport functors on configuration space and on parameter space...	$[\mathrm{tar},\mathrm{phas}] \xrightarrow{\mathrm{ev}^*} [\mathrm{conf}\times\mathrm{par},\mathrm{phas}]$, $\quad [\mathrm{conf}\times\mathrm{par},\mathrm{phas}] \xrightarrow{\bar{p_1^*}} [\mathrm{conf},\mathrm{phas}]$, $\quad [\mathrm{conf}\times\mathrm{par},\mathrm{phas}] \xrightarrow{\bar{p_2^*}} [\mathrm{par},\mathrm{phas}]$, $\quad [\mathrm{tar},\mathrm{phas}] \xrightarrow{\mathrm{t}} [\mathrm{conf},\mathrm{phas}]$, $\quad [\mathrm{tar},\mathrm{phas}] \xrightarrow{\mathrm{q}} [\mathrm{par},\mathrm{phas}]$
...that are known as the transgression and the quantization of the n-particle.	$\mathrm{tra} \overset{\text{transgression}}{\longmapsto} t(\mathrm{tra})$, $\quad \mathrm{tra} \overset{\text{quantization}}{\longmapsto} q(\mathrm{tra})$

Table 2: **The story of the charged n-particle.** A drama in three acts.

2.2 How to compute the space of n-sections

For computing the quantization, it is convenient to proceed as follows.

We take the product conf $\times$ par to be the adjoint to the internal hom. Then

$$[\mathrm{conf} \times \mathrm{par}, \mathrm{phas}] \simeq [\mathrm{conf}, [\mathrm{par}, \mathrm{phas}]]\,.$$

The image of tra : tar $\to$ phas under

$$\begin{array}{ccccc} [\mathrm{tar}, \mathrm{phas}] & \xrightarrow{\mathrm{ev}^*} & [\mathrm{conf} \times \mathrm{par}, \mathrm{phas}] & \xrightarrow{\sim} & [\mathrm{conf}, [\mathrm{par}, \mathrm{phas}]] \\ \mathrm{tar} & & \longmapsto & & \mathrm{tar}_* \end{array}$$

is simply postcomposition with tra:

$$\mathrm{tra}_* \;:\; \left(\mathrm{par} \underset{\gamma'}{\overset{\gamma}{\Downarrow}} \mathrm{tar} \right) \;\mapsto\; \left(\mathrm{par} \underset{\gamma'}{\overset{\gamma}{\Downarrow}} \mathrm{tar} \xrightarrow{\mathrm{tar}} \mathrm{phas} \right).$$

The fush-forward

$$[\mathrm{conf} \times \mathrm{par}, \mathrm{phas}] \xrightarrow{\bar{p_1^*}} [\mathrm{par}, \mathrm{phas}]$$

then corresponds to the push-forward

$$\begin{array}{ccccc} \mathrm{conf} & \xrightarrow{p} & \{\bullet\} & & \\ & & & & \\ [\mathrm{conf}, [\mathrm{par}, \mathrm{phas}]] & \xrightarrow{\bar{p^*}} & [\{\bullet\}, [\mathrm{par}, \mathrm{phas}]] & \xrightarrow{\sim} & [\mathrm{par}, \mathrm{phas}] \\ \mathrm{tar}_* & & \longmapsto & & q(\mathrm{tra}) \end{array}.$$

This $q(\mathrm{tra})$ is then defined by

$$\mathrm{Hom}(p^* f_\bullet, \mathrm{tra}_*) \simeq \mathrm{Hom}(f_\bullet, q(\mathrm{tra}))\,,$$

for any $f_\bullet : \bullet \to [\mathrm{par}, \mathrm{phas}]$.

In practice, it is usually sufficient to consider this equivalence for $f_\bullet = 1_\bullet$, the tensor unit in $[\{\bullet\}, [\mathrm{par}, \mathrm{phas}]]$.

This is what we shall do in the following example.

3 Our Example: the string on the classifying space of a 2-group

Here we regard a special case of a globular extended quantum field theory of a charged n-particle.

3.1 The setup: parameter space, target space, etc.

We set $n = 2$ and take parameter space to be

$$\mathrm{par} = \Sigma(\mathbb{Z})$$

which we draw like

$$\Sigma(\mathbb{Z}) = \{\bullet \to \bullet\}$$

and think of as modelling a string that stretches from something back to that something. We'll discover what that something can be as we proceed.

This string is taken to propagate on a target space

$$\mathrm{tar} = \Sigma(G_2)\,,$$

which is the 2-category obtained by thinking of a strict 2-group G_2 as a one-object 2-groupoid.

Noticing that an element

$$\gamma : \Sigma(\mathbb{Z}) \to \Sigma(G_2)$$

in configuration space

$$\mathrm{conf} = [\Sigma(\mathbb{Z}), \Sigma(G_2)]$$

is, when we send it to the world of topological spaces by taking nerves and geometric realizations

$$|\gamma| : |\Sigma(\mathbb{Z})| \to |\Sigma(G_2)|$$

a based loop in the classifying space of G_2:

$$|\gamma| : S^1_\bullet \to BG_{2\bullet}\,,$$

we say that conf models the configurations of a string that propagates on the classifying space of a strict 2-group.

Notice, however, that this description is somewhat imprecise. In fact, our globular formalism remembers the difference between a circular open string and a closed string. This point will be addressed in detail later on.

All that remains to be specified, now, is the 2-bundle with connection on target space that we wish to couple our 2-particle to. For the moment, we shall be content with understanding the simple case where this 2-bundle is trivial, of rank one and with trivial connection.

This means that we take

$$\mathrm{tra} = 1 : \Sigma(G_2) \to \mathrm{Bim} \hookrightarrow 2\mathrm{Vect}$$

to be the tensor unit in the 2-category of all such 2-functors:

$$1 \;:\; \bullet \underset{g'}{\overset{g}{\Downarrow h}} \bullet \;\mapsto\; \mathbb{C} \underset{\mathbb{C}}{\overset{\mathbb{C}}{\Downarrow \mathrm{Id}}} \mathbb{C} \;.$$

3.2 Quantization: sections and push-forward to the point

Quantization of the above system amounts to finding

$$q(\mathrm{tra}_*) : \{\bullet\} \to [\Sigma(\mathbb{Z}), \mathrm{Bim}]$$

such that

$$\mathrm{Hom}_{[\mathrm{conf},[\mathrm{par},\mathrm{phas}]]}(1_*, \mathrm{tra}_*) \simeq \mathrm{Hom}_{[\{\bullet\},[\mathrm{par},\mathrm{phas}]]}(1, q(\mathrm{tra}_*)) \,.$$

We call

$$\Gamma(\mathrm{tra}_*) = \mathrm{Hom}(1_*, \mathrm{tra}_*)$$

the space of sections of the n-bundle on configuration space.

Remember that we want to concentrate on $\mathrm{tra}_* = 1_*$.

As we have shown elsewhere, we find

Proposition 2

$$\Gamma(1_*) = \Lambda\mathrm{Rep}(\Lambda G_2) \,.$$

Here

$$\Lambda G_2 = \mathrm{conf}/\sim$$

is the **loop groupoid** of G_2 obtained by identifying isomorphic 1-morphisms in $\mathrm{conf} = [\Sigma(\mathbb{Z}), \Sigma(G_2)]$. This is a slight generalization of Willerton's loop groupoid.

Moreover

$$\mathrm{Rep}(\Lambda G_2) = [\Lambda G_2, \mathrm{Vect}]$$

is the category of representations of the loop groupoid and

$$\Lambda\mathrm{Rep}(\Lambda G_2) = [\Sigma(\mathbb{Z}), \mathrm{Rep}(\Lambda G_2)]$$

is the category of loops inside the category of representations of the loop groupoid. An object in there is an automorphism

$$\begin{array}{c} \rho \\ \downarrow{\scriptstyle L} \\ \rho \end{array}$$

of a representation ρ of ΛG_2.

The quantization $q(1_*)$ that we are after will have to be a functor on the point, with an equivalent space of sections.

We find that, up to equivalence,

$$q(1_*) : \{\bullet\} \to [\mathrm{par}, \mathrm{phas}]$$

is given by

$$q(1_*)(\bullet) = (\mathbb{C}[\Lambda G_2] \stackrel{\mathrm{Id}}{\to} \mathbb{C}[\Lambda G_2])\,,$$

where $\mathbb{C}[\Lambda G_2]$ denotes the groupoid algebra of ΛG_2.

Notice that under the embedding

$$\mathrm{Bim} \hookrightarrow 2\mathrm{Vect}$$

we have

$$\mathbb{C}[\Lambda G_2] \mapsto \mathrm{Mod}_{\mathbb{C}[\Lambda G_2]} \simeq \mathrm{Rep}(\Lambda G_2)\,.$$

A section of $q(1_*)$

$$e : 1_\bullet \to q(1_*)$$

is a square

$$\begin{array}{ccc} \mathbb{C} & \xrightarrow{\mathrm{Id}} & \mathbb{C} \\ {\scriptstyle e_\bullet}\downarrow & \swArrow e_{\bullet\to\bullet} & \downarrow{\scriptstyle e_\bullet} \\ \mathbb{C}[\Lambda G_2] & \xrightarrow[\mathrm{Id}]{} & \mathbb{C}[\Lambda G_2] \end{array}$$

in Bim. Notice that this means that $e_\bullet$ is a module for $\mathbb{C}[\Lambda G]$ and that $e_{\bullet\to\bullet}$ is an automorphism of that representation.

This way we discover that, indeed

$$\mathrm{End}(1_*) \simeq [1, q(1_*)]\,.$$

$q(1_*)$ is the *quantization* of our 2-bundle on our target space.

In conclusion, we hence find that the globular extended QFT coming from the string propagating "on a 2-group" G_2 yields a propagation 1-functor on parameter space

$$q(1_*) : \mathrm{par} \to \mathrm{Bim} \hookrightarrow 2\mathrm{Vect}$$

of the form

$$q \;:\; (\,\bullet \longrightarrow \bullet\,) \;\mapsto\; (\,\mathbb{C}[\Lambda G_2] \xrightarrow{\mathrm{Id}} \mathbb{C}[\Lambda G_2]\,)\,.$$

3.3 Nontrivial topology: traces and states of closed string

A trace is what distiguishes a circular path from a circle.

A trace is what distinguishes a circular open string from a closed string.

Gluing. We need a trace to glue the ends of

$$
\begin{array}{c}
\rho \\
\downarrow{\scriptstyle L} \\
\rho
\end{array}
\,.
$$

This gives

$$
\mathrm{Tr}
\begin{array}{c}
\rho \\
\downarrow{\scriptstyle L} \\
\rho
\end{array}
\;=\;
\begin{array}{ccccc}
 & \swarrow & 1 & \nwarrow & \\
 & \rho & & \rho^* & \\
{\scriptstyle L} & \downarrow & & \uparrow & {\scriptstyle \mathrm{Id}} \\
 & \rho & & \rho^* & \\
 & \searrow & 1 & \nearrow &
\end{array}
\,,
$$

where 1 denotes the tensor unit in $\mathrm{Rep}(\Lambda G_2)$, that is the trivial 1-dimensional representation.

The result of the trace is an endomorphism of the trivial representation:

$$
\mathrm{Tr} : \Lambda(\mathrm{Rep}\Lambda G_2) \to \mathrm{End}_{1_{\mathrm{Rep}\Lambda G_2}} \,.
$$

Notice that an endomorphism of the trivial representation of a groupoid is a function on the set of its connected components.

States of open and of closed strings. In conclusion, we find that our globular extended QFT assigns to the open string states that are objects in the category

$$
\Lambda \mathrm{Rep}(\Lambda \mathrm{G}_2)\,.
$$

An object in here

$$
\begin{array}{c}
\rho \\
\downarrow{\scriptstyle L} \\
\rho
\end{array}
$$

can be thought of as the state of an open string that sits on a "ρ-brane". (Indeed, as discussed elsewhere, for the case that $G_2 = \mathrm{String}_G$, representations ρ of ΛG_2 correspond to G-equivariant modules of the canonical gerbe on G, hence to D-branes on G in the familiar sense.)

To the closed string, on the other hand, it assigns states that are objects in

$$
\mathrm{Tr}(\Lambda \mathrm{Rep}(\Lambda \mathrm{G}_2)) = \mathrm{End}_{1_{\mathrm{Rep}\Lambda G_2}} \,,
$$

which is the space of functions on connected components of ΛG_2.

3.4 The disk correlator

Given a quantum state of our string on $\Sigma(G_2)$

$$e_1 : 1 \to q(\mathrm{tra}_*)$$

and given a costate

$$\bar{e}_2 : q(\mathrm{tra}_*) \to 1$$

we may pair both in a canonical fashion to obtain

$$(\bar{e}_2, e_1) \ : \ 1 \xrightarrow{e_1} q(\mathrm{tra}_*) \xrightarrow{\bar{e}_2} 1 \ .$$

This lives in $\mathrm{End}(1)$, which itself is monoidal. So we may choose a second order section

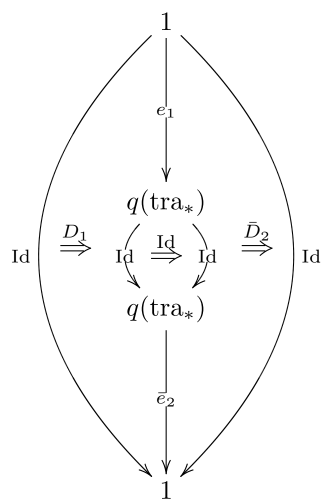
$$D_1 : \mathrm{Id}_1 \to (\bar{e}_2, e_1)$$

and a second order cosection

$$\bar{D}_2 : (\bar{e}_2, e_1) \to \mathrm{Id}_1$$

and form the second order pairing:

$(\bar{D}_2, D_1) \quad =$



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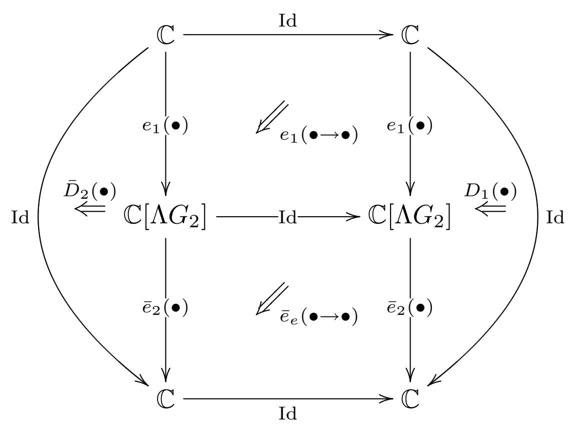
As explained elsewhere, this may be addressed as a disk correlator in our QFT.

Concretely, this is a modification of transformations of 2-functors, whose single component is the 2-cell



in Bim.

For simplicity, consider the example where

$$e_1(\bullet) = N$$

is any right $\mathbb{C}[\Lambda G_2]$-module and where

$$\bar{e}_2(\bullet) = N^\vee$$

is the corresponding dual left $\mathbb{C}[\Lambda G_2]$-module.

Then the duality on these provides canonical choices for D_1 and $\bar{D}_2$.

If we further abbreviate

$$A := \mathbb{C}[\Lambda G_2]$$

and

$$L := {}_{\mathbb{C}}A_A$$

for A regarded as a right A-module over itself and

$$R := {}_A A_{\mathbb{C}}$$

as A regarded as a left A-module over itself, then, as discussed elsewhere, the above 2-cell in Bim may equivalently be rewritten – after applying local trivialization and after passing from globular to dual string diagrams – as