1 Introduction

In this article we shall briefly outline derived categories and their relevance for physics. Derived categories (and their enhancements) classify off-shell states in a two-dimensional topological field theory on Riemann surfaces with boundary known as the open string B model. We briefly review pertinent aspects of that topological field theory and its relation to derived categories, the Bondal-Kapranov enhancement and its relation to the open string B model, as well as B model twists of two-dimensional theories known as Landau-Ginzburg models, and how information concerning stability of D-branes is encoded in this language. We concentrate on more physical aspects of derived categories; for a very readable short review concentrating on the mathematics, see *e.g.* [8].

2 Sheaves and derived categories in the open string B model

Derived categories are mathematical constructions which are believed to be related to D-branes in the open string B model. We shall begin by briefly reviewing the B model, as well as D-branes.

The A and B models are two-dimensional topological field theories, closely related to nonlinear sigma models, which are supersymmetrizations of theories summing over maps from a Riemann surface (the worldsheet of the string) into some "target space" X. In both the A and B model one considers only certain special correlation functions, involving correlators closed under the action of a nilpotent scalar operator known as the "BRST operator, " Q, which is part of the original supersymmetry transformations. In considering the pertinent correlation functions, only certain types of maps contribute. The A model has the properties of being invariant under complex structure deformations of the target space X, and its pertinent correlation functions are computed by summing over holomorphic maps into the target X. The A model will not be relevant for us here. The B model has the properties of being invariant under Kähler moduli of the target X, and its pertinent correlation functions are computed by summing over constant maps into the target X. In the closed string B model, the states of the theory are counted by the cohomology groups $H^*(X, \Lambda^* TX)$, where X is constrained to be Calabi-Yau. The BRST operator in the B model Q can be identified with $\overline{\partial}$ for many purposes. The open string B model is the same topological field theory, but now defined on a Riemann surface with boundaries. As with all open string theories, we specify boundary conditions on the fields, which force the ends of the string to live on some submanifold of the target, and we associate to the boundaries degrees of freedom (known as the Chan-Paton factors) which describe a (possibly twisted) vector bundle over the submanifold. In the case of the B model, the submanifold is a complex submanifold, and the vector bundle is forced to be a holomorphic vector bundle over that submanifold.

To lowest order, that combination of a submanifold S of X together with a (possibly twisted) holomorphic vector submanifold, is a "D-brane" in the open string B model. We shall denote the twisted bundle by $\mathcal{E} \otimes \sqrt{K_S}$, where K_S is the canonical bundle of S, and the $\sqrt{K_S}$ factor is an explicit incorporation of something known as the the Freed-Witten anomaly. Now, if $i : S \hookrightarrow X$ is the inclusion map, then to this D-brane we can associate a sheaf $i_* \mathcal{E}$.

Technically, a sheaf is defined by associating sets, or modules, or rings, to each open set on the underlying space, together with restriction maps saying how data associated to larger open sets restricts to smaller open sets, obeying the obvious consistency conditions, together with some gluing conditions that say how local sections can be patched back together. A vector bundle defines a sheaf, by associating to any open set sections of the bundle over that open set. Sheaves of the form "$i_* \mathcal{E}$" look like, intuitively, vector bundles over submanifolds, with vanishing fibers off the submanifold. A more detailed discussion of sheaves is beyond the scope of this article; see instead *e.g.* [7].

To "associate a sheaf" means finding a sheaf such that physical properties of the D-brane system are well-modelled by mathematics of the sheaf. (In particular, the physical definition of D-brane has, on the face of it, nothing at all to do with the mathematical definition of a sheaf, so one cannot directly argue that they are the same, but one can still use one to give a mathematical model of the other.) For example, the spectrum of open string states in the B model stretched between two D-branes, associated to sheaves $i_* \mathcal{E}$ and $j_* \mathcal{F}$, turn out to be calculated by a cohomology group known as $\mathrm{Ext}^*_X(i_* \mathcal{E}, j_* \mathcal{F})$.

There are many more sheaves not of the form $i_* \mathcal{E}$, that is, that do not look like vector bundles over submanifolds. It is not known in general whether they also correspond to (on-shell) D-branes, but in some special cases the answer has been worked out. For example, structure sheaves of nonreduced schemes turn out to correspond to D-branes with nonzero nilpotent Higgs vevs.

For a set of ordinary D-branes, the description above suffices. However, more generally one would like to describe collections of D-branes and anti-D-branes, and tachyons. An anti-D-brane has all the same physical properties as an ordinary D-brane, modulo the fact that they try to annihilate each other. The open string spectrum between coincident D-branes and anti-D-branes contains tachyons. One can give an (off-shell) vacuum expectation value to such tachyons, and then the unstable brane-antibrane-tachyon system will evolve to some other, usually simpler, configuration. For example, given a single D-brane wrapped on a curve, with trivial line bundle, and an anti-D-brane wrapped on the same curve, with line bundle $\mathcal{O}(-1)$, and a nonzero tachyon $\mathcal{O}(-1) \rightarrow \mathcal{O}$, then one expects that the system will dynamically evolve to a smaller D-brane sitting at a point on the curve.

Now, one would like to find some mathematics that describes such systems, and gives information about the endpoints of their evolution. Technically, one would like to classify universality classes of worldsheet boundary renormalization group flow.

It has been conjectured that derived categories of sheaves provide such a classification. To properly explain derived categories is well beyond the scope of this article (see instead the reading list at the end), but we shall give a short outline below.

Mathematically, derived categories of sheaves concern complexes of sheaves, that is, sets of sheaves $\mathcal{E}_i$ together with maps $d_i : \mathcal{E}_i \to \mathcal{E}_{i+1}$

$$\cdots \longrightarrow \mathcal{E}_i \xrightarrow{d_i} \mathcal{E}_{i+1} \xrightarrow{d_{i+1}} \mathcal{E}_{i+2} \xrightarrow{d_{i+2}} \cdots$$

such that $d_{i+1} \circ d_i = 0$. A category is defined by a collection of 'objects' together with maps between the objects, known as morphisms. In a derived category of coherent sheaves, the objects are complexes of sheaves, and the maps are equivalence classes of maps between complexes.

Physically, if the complex consists of locally-free sheaves (equivalently, vector bundles), then we can associate a brane/antibrane/tachyon system, by identifying the $\mathcal{E}_i$ for i even, say, with D-branes, and the $\mathcal{E}_i$ for i odd with anti-D-branes. If the $\mathcal{E}_i$ are all locally-free sheaves, then there are tachyons between the branes and antibranes, and we can identify the d_i's with those tachyons. In the open string worldsheet theory, giving a tachyon a vacuum expectation value modifies the BRST operator Q, and a necessary condition for the new theory to still be a topological field theory is that $Q^2 = 0$, a condition which turns out to imply that $d_{i+1} \circ d_i = 0$.

To recreate the structure of a derived category, we need to impose some equivalence relations. To see what sorts of equivalence relations one would like to impose, note the following. Physically, we would like to identify, for example, a configuration consisting of a brane, an antibrane, and a tachyon, which we can describe as a complex

$$\mathcal{O}(-D) \longrightarrow \mathcal{O}$$

with a one-element complex

$$\mathcal{O}_D$$

corresponding to the D-brane which we believe is the endpoint of the evolution of the brane/antibrane configuration.

One natural mathematical way to create identifications of this form, is to identify complexes that differ by "quasi-isomorphisms," meaning, a set of maps ($f^n : C^n \to D^n$) compatible with d's, and inducing an isomorphism $\tilde{f}^n : H^n(C) \cong H^n(D)$ on the cohomologies of the complexes. In particular, in the example above, there is a natural set of maps

$$\begin{tikzcd} \mathcal{O}(-D) \arrow[r] \arrow[d] & \mathcal{O} \arrow[d] \\ 0 \arrow[r] & \mathcal{O}_D \end{tikzcd}$$

that define a quasi-isomorphism. More generally, in homological algebra one typically does computations by replacing ordinary objects with projective or injective resolutions, *i.e.* complexes with special properties, in which the desired computation becomes trivial, and defining the result for the original object to be the same as the result for the resolution. To formalize this procedure, one would like a mathematical setup in which objects and their projective and injective resolutions are isomorphic.

However, to define an equivalence relation, one usually needs an isomorphism, and the quasi-isomorphisms above are not, in general, isomorphisms. Creating an equivalence from non-isomorphisms, to resolve this problem, can be done through a process known as "localization" (generalizing the notion of localization in commutative algebra). The resulting equivalence relations on maps between complexes defines the derived category.

The derived category is a category whose objects are complexes, and whose morphisms $C^\cdot \to D^\cdot$ are equivalence classes of pairs (s, t) where $s : G^\cdot \to C^\cdot$ is a quasi-isomorphism between $C^\cdot$ and another complex $G^\cdot$, and $t : G^\cdot \to D^\cdot$ is a map of complexes. We take two such pairs (s, t), (s', t') to be equivalent if there exists another pair (r, h) between the auxiliary complexes $G^\cdot$, $G^{\cdot\prime}$, making the obvious diagram commute. This is, in a nutshell, what is meant by localization, and by working with such equivalence classes, this allows us to formally invert maps that are otherwise non-invertible. (We encourage the reader to consult the references for more details.)

Mathematically, this technology gives a very elegant way to rethink *e.g.* homological algebra. There is a notion of a derived functor, a special kind of functor between derived categories, and notions from homological algebra such as Ext and Tor can be re-expressed as cohomologies of the image complexes under the action of a derived functor, thus replacing cohomologies with complexes.

Physically, looking back at the physical realization of complexes, we see a basic problem: different representatives of (isomorphic) objects in the derived category are described by very different physical theories. For example, the sheaf $\mathcal{O}_D$ corresponds to a single D-brane, defined by a two-dimensional boundary conformal field theory, whereas the brane/antibrane/tachyon collection $\mathcal{O}(-D) \to \mathcal{O}$ is defined by a massive non-conformal two-dimensional theory. These are very different physical theories. If we want "localization on quasi-isomorphisms" to happen in physics, we have to explain which properties of the physical theories we are interested in, because clearly the entire physical theories are not and cannot be isomorphic.

Although the entire physical theories are not isomorphic, we can hope that under renormalization group flow, the theories will become isomorphic. That is certainly the physical content of the statement that the brane/antibrane system $\mathcal{O}(-D) \rightarrow \mathcal{O}$ should describe the D-brane corresponding to $\mathcal{O}_D$ – after worldsheet boundary renormalization group flow, the nonconformal two-dimensional theory describing the brane/antibrane system becomes a conformal field theory describing a single D-brane.

More globally, this is the general prescription for finding physical meanings of many categories: we can associate physical theories to particular types of representatives of isomorphism classes of objects, and then although distinct representatives of the same object may give rise to very different physical theories, those physical theories at least lie in the same universality class of worldsheet renormalization group flow. In other words, (equivalence classes of) objects are in one-to-one correspondence with universality classes of physical theories.

Showing such a statement directly is usually not possible – it is usually technically impracticle to follow renormalization group flow explicitly. There is no symmetry reason or other basic physics reason why renormalization group flow must respect quasi-isomorphism. The strongest constraint that is clearly applied by physics is that renormalization group flow must preserve D-brane charges (Chern characters, or more properly, K-theory), but objects in a derived category contain much more information than that.

However, indirect tests can be performed, and because many indirect tests are satisfied, the result is generally believed.

The reader might ask why it is not more efficient to just work with the cohomology complexes $H^{\cdot}(C)$ themselves, rather than the original complexes. One reason is that the original complexes contain more information than the cohomology – passing to cohomology loses information. For example, there exist examples of complexes that have the same cohomology, yet are not quasi-isomorphic, and so are not identified within the derived category, and so physically are believed to lie in different universality classes of boundary RG flow.

Another motivation for relating physics to derived categories is Kontsevich's approach to mirror symmetry. Mirror symmetry relates pairs of Calabi-Yau manifolds, of the same dimension, in a fashion such that easy classical computations in one Calabi-Yau are mapped to difficult 'quantum' computations involving sums over holomorphic curves in the other Calabi-Yau. Because of this property, mirror symmetry has proven a fertile ground for algebraic geometers to study. Kontsevich proposed that mirror symmetry should be understood as a relation between derived categories of coherent sheaves on one Calabi-Yau and derived Fukaya categories on the other Calabi-Yau. At the time he made this proposal, no one had any idea how either could be realized in physics, but since that time, physicists have come to believe that Kontsevich was secretly talking about D-branes in the A and B models.

Figure 1: Example of generalized complex. Each arrow is labelled by the degree of the corresponding vertex operator.

3 Bondal-Kapranov enhancements

Mathematically derived categories are not quite as ideal as one would like. For example, the cone construction used in triangulated categories, does not behave functorially – the cone depends upon the representative of the equivalence class defining an object in a derived category, and not just the object itself.

Physically, our discussion of brane/antibrane systems was not the most general possible. One can give vacuum expectation values to more general vertex operators, not just the tachyons.

Curiously, these two issues solve each other. By incorporating a more general class of boundary vertex operators, one realizes a more general mathematical structure, due to Bondal and Kapranov, which repairs many of the technical deficiencies of ordinary derived categories. Ordinary complexes are replaced by generalized complexes in which arrows can map between non-neighboring elements of the complex. Schematically, the BRST operator is deformed by boundary vacuum expectation values to the form

$$Q \;=\; \overline{\partial} \,+\, \sum_a \phi_a$$

and demanding that the BRST operator square to zero implies that

$$\sum_a \overline{\partial} \phi_a \;+\; \sum_{a,b} \phi_b \circ \phi_a \;=\; 0$$

which is the same as the condition for a generalized complex. Note that for ordinary complexes, the condition above factors into

$$\begin{aligned} \overline{\partial} \phi_n &= 0 \\ \phi_{n+1} \circ \phi_n &= 0 \end{aligned}$$

which yields an ordinary complex of sheaves.

4 Landau-Ginzburg models

So far we have described how derived categories are relevant to geometric compactifications, *i.e.* sigma models on Calabi-Yau manifolds. However, there are also “nongeometric” theories

– CFT's that do not come from sigma models on manifolds, of which Landau-Ginzburg models and their orbifolds are prominent examples. Landau-Ginzburg models can also be twisted into topological field theories, and the B-type topological twist of (an orbifold of) a Landau-Ginzburg model is believed to be isomorphic, as a topological field theory, to the B model obtained from a nonlinear sigma model, of the form we outlined earlier. Landau-Ginzburg models have a very different form than nonlinear sigma models, and so sometimes there can be practical computational advantages to working with one rather than the other.

A Landau-Ginzburg model is an ungauged sigma model with a nonzero superpotential (a holomorphic function over the target space that defines a bosonic potential and Yukawa couplings). (In 'typical' cases, the target space is a vector space.) Because of the superpotential, a Landau-Ginzburg model is a massive theory – not itself a CFT, but many Landau-Ginzburg models are believed to flow to CFT's under the renormalization group.

In formulating open strings based on Landau-Ginzburg models, naive attempts fail because of something known as the Warner problem: if the superpotential is nonzero, then the obvious ways to try to define the theory on a Riemann surface with boundary have the undesirable property that the supersymmetry transformations only close up to a nonzero boundary term, proportional to derivatives of the superpotential. In order to find a description of open strings in which the supersymmetry transformations close, one must take a very nonobvious formulation of the boundary data. Specifically, to solve the Warner problem, one is led to work with pairs of matrices whose product is proportional to the superpotential.

This method of solving the Warner problem is known as matrix factorization, and D-branes in this theory are defined by the factorization chosen, *i.e.* the choice of pairs of matrices. In simple cases, we can be more explicit as follows. Choose a set of polynomials F_α, G_α such that the Landau-Ginzburg superpotential W is given by

$$W \;=\; \sum_\alpha F_\alpha G_\alpha \;+\; \text{constant}$$

The F_α and G_α are used to define the boundary action – the F's appear as part of the boundary superpotential and the G's appear as part of the supersymmetry transformations of boundary fermi multiplets. The F_α and G_α, *i.e.* the factorization of W, determine the D-brane in the Landau-Ginzburg theory. We can also think of having a pair of holomorphic vector bundles $\mathcal{E}_1$, $\mathcal{E}_2$ of the same rank, and interpret F and G as holomorphic sections of $\mathcal{E}_1^\vee \otimes \mathcal{E}_2$ and $\mathcal{E}_2^\vee \otimes \mathcal{E}_1$, respectively, obeying $FG \propto W \cdot \mathrm{Id}$ and $GF \propto W \cdot \mathrm{Id}$, up to additive constants.

Although a Landau-Ginzburg model is not the same thing as a sigma model on a Calabi-Yau, orbifolds of Landau-Ginzburg models are often on the same Kähler moduli space. Perhaps the most famous example of this relates sigma models on quintic hypersurfaces in $\mathbf{P}^4$ to a $\mathbf{Z}_5$ orbifold of a Landau-Ginzburg model over $\mathbf{C}^5$ with five chiral superfields x_1, x_2, x_3, x_4, x_5, and a superpotential of the form

$$W \;=\; x_1^5 \,+\, x_2^5 \,+\, x_3^5 \,+\, x_4^5 \,+\, x_5^5 \,+\, \psi x_1 x_2 x_3 x_4 x_5$$

for ψ a complex number, corresponding to the equation of the degree-five hypersurface in $\mathbf{P}^4$. The (complexified) Kähler moduli space in this example is a $\mathbf{P}^1$, with the sigma model on the quintic at one pole, the zero-volume limit of the sigma model along the equator, and the Landau-Ginzburg orbifold at the opposite pole.

Since the closed string topological B model is independent of Kähler moduli, and the sigma model on the quintic and the Landau-Ginzburg orbifold above lie on the same Kähler moduli space, one would expect them both to have the same spectrum of D-branes, and indeed this is believed to be true.

5 Pi-stability

So far we have talked about D-branes in the topological B model, a topological twist of a physical sigma model. If we untwist back to a physical sigma model, then the stability of those D-branes becomes an issue.

To begin to understand what we mean by stability in this context, consider a set of N D-branes wrapped on, say, a K3 surface, at large radius (so that worldsheet instanton corrections are small). On the worldvolume of the D-branes, we have a rank N vector bundle, and in the physical theory on that worldvolume we have a consistency condition for supersymmetric vacua, that the vector bundle be "Mumford-Takemoto stable." To understand what is meant by this condition on a Kähler manifold, let ω denote the Kähler form, and define the "slope" μ of a vector bundle $\mathcal{E}$ on a manifold X of complex dimension n to be given by

$$\mu(\mathcal{E}) \; = \; \frac{\int_X \omega^{n-1} \wedge c_1(\mathcal{E})}{\mathrm{rank}\; \mathcal{E}}$$

where ω is the Kähler form. Then, we say that $\mathcal{E}$ is (semi-)stable if for all subsheaves $\mathcal{F}$ satisfying certain consistency conditions, $\mu(\mathcal{F})(\leq) < \mu(\mathcal{E})$.

Since the slope of a bundle depends upon the Kähler form, whether a given bundle is Mumford-Takemoto stable depends upon the metric. In general, on a Kähler manifold, the Kähler cone breaks up into subcones, with a different moduli space of (stable) holomorphic vector bundles in each subcone.

This is a mathematical notion of stability, but it also corresponds to physical stability, at least in a regime in which quantum corrections are small. If a given bundle is only stable in a proper subset of the Kähler cone, then when it reaches the boundary of the subcone in which it is stable, the gauge field configuration that satisfies the Donaldson-Uhlenbeck-Yau partial differential equation splits into a sum of two separate bundles. In a heterotic string compactification, this leads to a low-energy enhanced $U(1)$ gauge symmetry and D-terms which realize the change in moduli space. In D-branes, this means the formerly bound state of D-branes (described by an irreducible holomorphic vector bundle) becomes only marginally bound; a decay becomes possible.

Pi-stability is a proposal for generalizing the considerations above to D-branes no longer wrapping the entire Calabi-Yau, and including quantum corrections.

In order to define pi-stability, we must first introduce a notion of grading φ of a D-brane. Specifically, for a D-brane wrapped on the entire Calabi-Yau X with holomorphic vector bundle $\mathcal{E}$, the grading is defined as the mirror to the expression $\int_X \mathrm{ch}(\mathcal{E}) \wedge \Pi$, where Π encodes the periods. Close to the large-radius limit, this has the form:

$$\varphi(\mathcal{E}) \; = \; \frac{1}{\pi} \mathrm{Im} \; \log \int_X \exp\left(B + i\omega\right) \wedge \mathrm{ch}\left(\mathcal{E}\right) \wedge \sqrt{\mathrm{td}\left(TX\right)} \; + \; \cdots$$

where B is a two-form, the "B field." As defined φ is clearly S^1-valued; however, we must choose a particular sheet of the log Riemann surface, to obtain a $\mathbf{R}$-valued function.

This notion of grading of D-branes is an ansatz, introduced as part of the definition of pi-stability. Physically, it is believed that the difference in grading between two D-branes corresponds to the fractional charge of the boundary-condition-changing vacuum between the two D-branes, though we know of no convincing first-principles derivation of that statement. In particular, unlike closed string computations, the degree of the Ext group element corresponding to a particular boundary R-sector state is not always the same as the $U(1)_R$ charge – for example, it is often determined by the $U(1)_R$ charge *minus* the charge of the vacuum. The grading gives us the mathematical significance of that vacuum charge. This mismatch between Ext degrees and $U(1)_R$ charges is necesary for the grading to make sense: Ext groups degrees are integral, after all, yet we want the grading to be able to vary continuously, so the grading had better not be the same as an Ext group degree.

Given a $\mathbf{R}$-valued function from a particular definition of log in the definition of φ above, the statement of pi-stability is then that for all subsheaves $\mathcal{F}$, as in the statement of Mumford-Takemoto stability,

$$\varphi(\mathcal{F}) \; \leq \; \varphi(\mathcal{E})$$

Before trying to understand the physical meaning of φ, or the extension of these ideas to derived categories, let us try to confirm that Mumford-Takemoto stability emerges as a limit of pi-stability.

For simplicity, suppose that X is a Calabi-Yau threefold. Then, for large Kähler form ω, we can expand $\varphi(\mathcal{E})$ as,

$$\varphi(\mathcal{E}) \; \approx \; \frac{1}{\pi} \mathrm{Im} \; \log \left[- \frac{i}{3!} \int_X \omega^3 (\mathrm{rk} \; \mathcal{E}) \right] \; + \; \frac{3}{\pi} \frac{\int_X \omega^2 \wedge c_1(\mathcal{E})}{\int_X \omega^3 \left(\mathrm{rk} \; \mathcal{E}\right)} \; + \; \cdots$$

Thus, we see that to leading order in the Kähler form ω, $\varphi(\mathcal{F}) \leq \varphi(\mathcal{E})$ if and only if

$$\frac{\int_X \omega^2 \wedge c_1(\mathcal{F})}{\text{rk } \mathcal{F}} \leq \frac{\int_X \omega^2 \wedge c_1(\mathcal{E})}{\text{rk } \mathcal{E}}$$

which is precisely the statement of Mumford-Takemoto stability on a threefold X.

One can define a notion of (classical) stability for more general sheaves, but what one wants is to apply pi-stability to derived categories, not just sheaves.

However, there is a technical problem that limits such an extension. Specifically, in a derived category there is no meaningful notion of "suboject." Thus, a notion of stability formulated in terms of subobjects cannot be immediately applied to derived categories. There are two (equivalent) workarounds to this issue that have been discussed in the math and physics literatures, which can be briefly summarized as follows:

1. One workaround involves picking a subcategory of the derived category that does allow you to make sense of subobjects. Such a structure is known, loosely, as a "T-structure," and so one can imagine formulating stability by first picking a T-structure, then specifying a slope function on the elements of the subcategory picked out by the subcategory.
2. Another (equivalent) workaround is to work with a notion of "relative stability." Instead of speaking about whether a D-brane is stable against decay into any other object, one only speaks about whether it is stable against decay into pairs of specified objects.

In this fashion, one can make sense of pi-stability for derived categories.

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