

On the Canonical Quantization of the 1-Particle

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What you are seeing here

The following is currently nothing but a few sketchy remarks supposed to illustrate a discussion given elsewhere in some blog entry on n -functorial quantum theory.

As time permits, it might evolve into something self-contained eventually. Or it might not.

Plan

- 1 The charged n -Particle.
- 2 Sections and Quantum States of the n -Particle.
- 3 Pull-Push Propagation of States of the n -Particle.
- 4 Example: The charged 1-particle on a lattice.

What on Earth is a “charged n -particle”?

See elsewhere.

Associated Sections

Definition

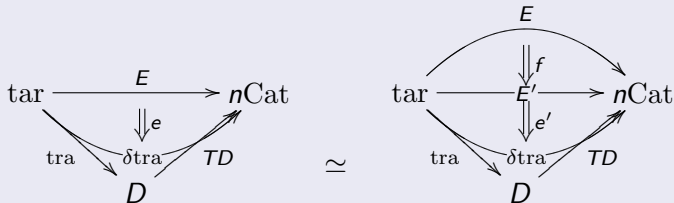
Given a parallel transport n -functor $\text{tra} : \text{tar} \rightarrow D$, its (“associated”) sections are transformations

$$\begin{array}{ccc}
 \text{tar} & \xrightarrow{E} & n\text{Cat} \\
 \searrow \text{tra} & \Downarrow e & \nearrow TD \\
 & \delta\text{tra} & \\
 & D &
 \end{array}$$

Associated Sections

Definition

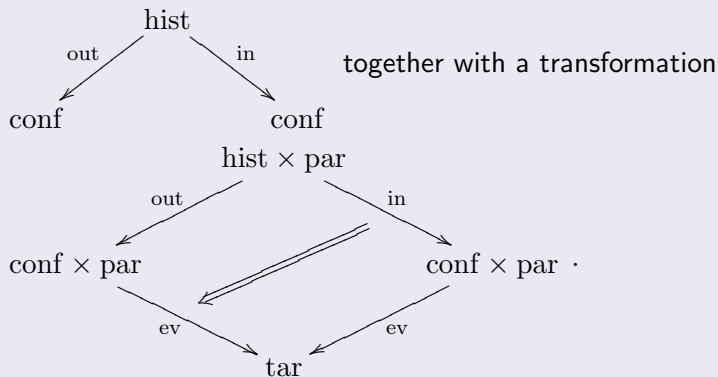
A morphism of sections $(E, e) \rightarrow (E', e')$ is a transformation $f : E \rightarrow E'$ together with an n -equivalence



The space of histories

Definition

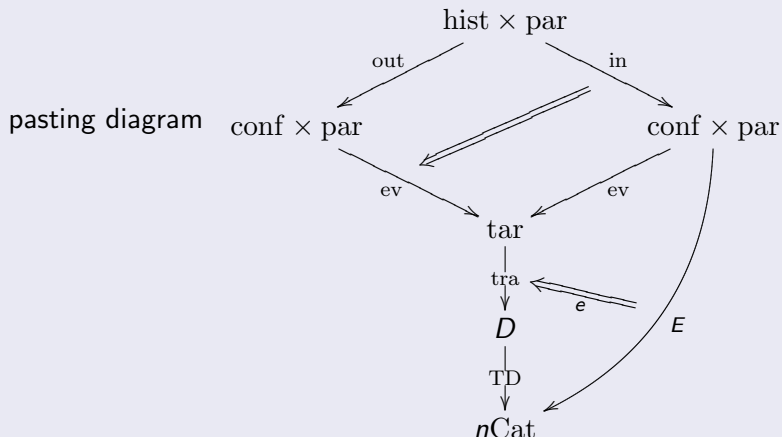
For conf the n -category of configurations of the n -particle, we say that an n -category of one-step histories is a span



Pasting a section to the space of histories

Definition

The pullback of a section through the space of histories is the



Pasting a section to the space of histories

Definition

The propagation of a section with respect to a given space of histories is the pushforward of the previous pullback along out.

The charged 1-particle on a lattice

The setup

The 1-particle on a lattice and charged under a trivial G -bundle is modeled by taking $\text{tar} = \mathbb{Z}/\mathbb{Z}$ and $\text{tra} : \text{tar} \rightarrow \Sigma G$.

We take $\text{hist} \subset \text{Hom}(a \rightarrow b, \text{tar})$ to be that subcategory whose objects come only from generators of tar . This expresses the fact that these are histories of “one time step”.

The charged 1-particle on a lattice

Remark

Notice that δ_{tra} takes values in $T_{\bullet}\Sigma G = \text{INN}(G) = G//G$.

In the combinatoric picture of linear algebra following the *Tale of Groupoidification* this corresponds to the regular representation space of G .

Therefore we will – probably – eventually, following the *Tale*, split idempotents and project out of that regular rep an irrep.

The charged 1-particle on a lattice

Remark

Consider, for simplicity, that we have a section (E, e) of tra where E factors through 0Cat . Then this is

- over each object x of target space a G -phased set $E(x) \rightarrow G \hookrightarrow G//G$.
- over each morphism of target space the covariant derivative with respect to tra of these G -phased sets.

$$\begin{array}{ccc}
 E(x) & \longrightarrow & E(y) \\
 \downarrow e(x) & \nearrow e(\gamma) & \downarrow e(y) \\
 G//G & \xrightarrow{\text{tra}(x \rightarrow y)^*} & G//G
 \end{array}$$

More generally we get G -phased groupoids.

The charged 1-particle on a lattice

Push-forward of 1-sections of the 1-particle corresponds to taking the coproduct of these G -phased sets.

The charged 1-particle on a lattice

If we can assume that functor underlying a transformation e coming from a section is *free* (I am not sure yet how this can be nicely implemented), then propagating a section along the space of one-step histories corresponds, unless I am mixed up, to the ordinary (euclidean, latticized in our example) quantum propagation.

While more needs to be done, clearly, it seems we find evidence that

- The measures needed when quantizing may arise canonically from colimits by expressing the path integral as a pull-push operation of states of an n -particle propagating on an n -categorical target space.
- Three mysteries are apparently solved in one stroke by this method: the funny shift in categorical dimension, the appearance of phases and the need to pass from principal to associated transport all come from the categorical differential δtra of the classical transport n -functor tra .