arrow-theory of amplimorphisms

February 5, 2007

Abstract

The arrow-theory underlying the original definition of "amplimorphisms" of algebras.

Let $\mathcal{C}$ be any abelian braided monoidal category.

Let $\mathrm{Bim}(\mathcal{C})$ be the 2-category whose objects are algebras internal to $\mathcal{C}$, whose morphisms are bimodules and whose 2-morphisms are bimodule homomorphism. Horizontal composition is the tensor product of bimodules.

(If we are restricting attention to *Frobenius* algebras we could choose to write $\mathrm{Frob}(\mathcal{C})$ instead.)

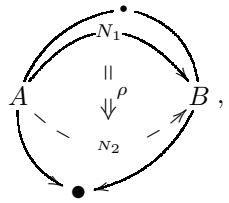
Since $\mathcal{C}$ is braided, $\mathrm{Bim}(\mathcal{C})$ is itself monoidal.

We write

$$\Sigma(\mathrm{Bim}(\mathcal{C}))$$

for the corresponding 3-category with a single object.

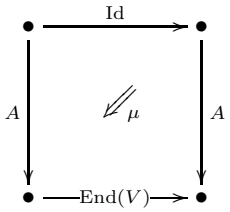
A 3-morphism in here looks like



where A and B are algebras, N_1 and N_2 are A-B-bimodules and ρ is a bimodule intertwiner.

We already know that of particular interest are categories of *cylinders* in $\Sigma(\mathrm{Bim}(\mathcal{C}))$.

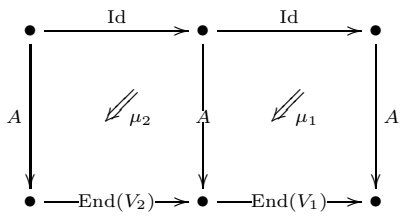
Consider cylinders in $\Sigma(\mathrm{Bim}(\mathcal{C}))$ of the form



and require that μ is a bimodule coming from a bimodule homomorphism

$$\mu : A \to A \otimes \mathrm{End}(V)\,.$$

These cylinders have an obvious horizontal composition

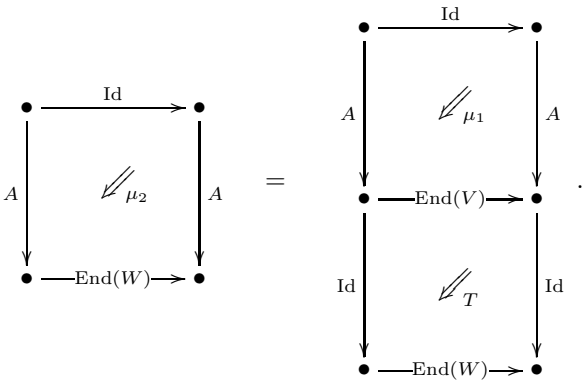


which translates into the obvious composition of the corresponding algebra homomorphisms:

$$\mu_2 \circ \mu_1 : A \to A \otimes \mathrm{End}(V_2) \otimes \mathrm{End}(V_1)\,.$$

The μ_i here have been called **amplimorphisms** of algebras.

Moreover, vertical composition of squares gives a notion of 2-morphisms of such amplimorphisms: let $T : \mu_1 \Rightarrow \mu_2$ be given by



.

This is manifestly compatible with the horizontal compositon of amplimorphisms in the obvious way.

If we assume $V \simeq W$ and let T be the bimodule coming from an algebra homomorphism of the form

$$\mathrm{Ad}_t : \mathrm{End}(V) \to \mathrm{End}(W)$$

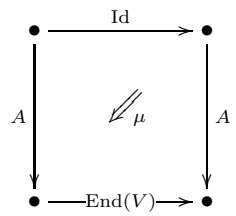
for any invertible $t \in \mathrm{Hom}(V, W)$, then such a T reproduces the notion of isomorphisms of amplimorphisms as given in the original literature.

(More details after lunch...)

An amplimorphism

$\mu : A \to A \otimes \mathrm{End}(V)$

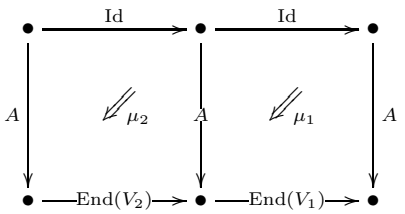
is a certain square in $\Sigma(\mathrm{Bim}(\mathcal{C}))$.



Composition of amplimorphisms

$\mu_2 \circ \mu_1 : A \to A \otimes \mathrm{End}(V_2) \otimes \mathrm{End}(V_1)$

is horizontal composition of these squares.



A morphism of amplimorphisms

$T : \mu_1 \to \mu_2$

is a tin can coming from vertical composition of such squares.

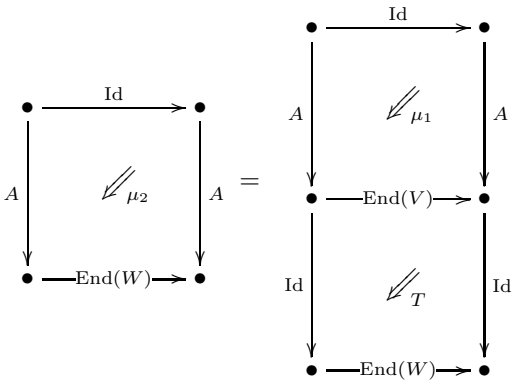


Figure 1: The **concept of amplimorphisms** in arrow-theoretical terms.