

Question in a unified theory of matter and space time:

Quantum physics

Geometry

↘ “Quantum geometry” ↙

- ⇒ Input from **Particle physics**
- ⇒ Input from **Cosmology**
- ⇒ Many developments in **Mathematical physics** and **Mathematics**, e.g.
 - Super-mathematics (super-algebra, super-geometry)
 - New cohomology theories (elliptic cohomology, quantum cohomology)
 - Special geometric structures (Kähler-, Einstein manifolds)
 - Infinite-dimensional algebraic structures (loop groups, vertex algebras ...)
 - Non-commutative geometry
 - ...

Vision for projects A4 – A7:

Contribute to a **physical** theory of matter and space time

Physical theories

↔

Mathematical structure

Particle physics, Cosmology

Geometry, Algebra

Project A6 Mathematical Aspects of String Compactifications

(Cortés, Louis, with Schomerus)

String theory (10 dimensions) → Models of Particle physics and cosmology

$$\text{Compactification } R^{3,1} \times M$$

Flat four-dimensional Minkowski space with *unbroken* supersymmetry ⇒

Calabi-Yau spaces

Cosmological observations ⇒ non-vanishing cosmological constant

- Include torsion in geometry → Scalar potential → Cosmological constant
- Stability → conserved supercurrent ⇒

Generalized Calabi-Yau spaces

Particle Physics:

Spontaneously broken supersymmetry, potential for scalar fields

(→ Particle physics: [A1] → Physics at colliders: [B1], [B2])

Cosmology:

Modifications of space time metrics (→ time dependent string backgrounds: [A2])

Higher gauge theory :

- String theory: Kalb-Ramond gauge field $B_{\mu\nu}$ (generalizing gauge fields A_μ)
- Global aspects (cf. instantons): Bundle gerbes

Bundle gerbes:

- Generalized geometry twisted by bundle gerbe [A6]
- Gauge theories of p -forms: part of supergravity theories [A7]
- Coupling to string worldsheet $\int_{\text{worldsheet}} Bdf$ [A4]

Project A4 Mathematical Foundations of String Theory

(Schweigert, with Richter and Schreiber)

- Gerbe holonomy \rightarrow Wess-Zumino term (defects, unorientable strings \rightarrow [A6])
- Holographic description of rational conformal field theories
 - Dualities, Brauer groups, generalized worldsheet geometries
 - Beyond rationality \rightarrow cosmological backgrounds (\rightarrow [A2])
- Worldsheet boundary: twisted gauge theory on D-brane (“gerbe module”)
(D-Branes \rightarrow [A5,A6])

Project A5 Algebraic aspects of D-Branes

(Fredenhagen, Schweigert, with Bahns and Brunetti)

Boundary effects in string theory and quantum field theory

- D branes \rightarrow in generalized geometries: [A6], time-dependent backgrounds \rightarrow [A2]
- Casimir effect \rightarrow Cosmological singularities (\rightarrow [C6,C7])

Combination of conformal field theory and algebraic field theory

Project A7 Pseudo-Riemannian Geometry and Supersymmetry

(Cortés, with Louis)

- Long term goal: geometric theory for supergravity (\rightarrow [A1] \rightarrow [B2])
- Geometric structures of supersymmetric field theories and their moduli spaces (\rightarrow [A6]) E.g. eight real supercharges: special geometries related by c -maps

Mathematical expertise \longleftrightarrow All users of QFT, in particular particle physicists

- Representation theory, Lie theory (Cortés, Schweigert)
- Algebraic topology (Richter)
- Noncommutative algebra and geometry (Bahns, Schweigert)
- Differential geometry, super-geometry (Cortés)