notes from a talk by
K.-H. Rehren

AdS/CFT: far away from infinity talk at MPI, Bonn, conference *The manifold geometries of QFT*, 1st July, 2008
Joint work with Longo

what QFT is all about

- operators on a Hilbert space,
- associated with points x or regions O of Lorenzian spaceztme
- commuting at spacelike distances
- covariant under unitary rep of spactime symmetry group

fields are operator valued distributions $f \mapsto \phi(f)$ supp f $\subset O$ More general $e^{i\phi(f)} \in$ von Neumann algebra $A(O)$

Motivation Puzzles related to

- bulk localization vs. boundary localization
- bulk degrees of freedom vs. boundary degrees of freedom
- spacetime dimensionality
- Objections against "algebraic holography"

  for instance:

  "It is absurd that a mere re-organization of the same algebra can describe physics in different dimensions. One can measure the dimension by entropy considerations."

  Will not answer these questions in detail here but consider simple examples to show that it is not as absurd as it may seem.

Rational QFT on AdS_2 An instructive class of models that can be treated rigorously $\Rightarrow$ resolution of various "paradoxes"

Geometry usual cartoon of AdS
$ds^2 \sim d\tau^2 - d\xi^2$
$(0 < \tau - \xi < \tau + \xi < 2\pi)$
$\sim dt^2 - dx^2$
$(t \in \mathbb{R}, x > 0)$ Minkowski halspace
$(\tan\frac{\tau\pm\xi}{2} = t \pm x)$
common covering space = strip
$(\tau \in \mathbb{R}, \xi \in [0, \pi])$

$SO(2,1) = PSL(2,\mathbb{R}) = \text{Moeb}$

The isometry group of AdS2 is $SO(2,1)$. It acts on the covering space of M_+ through simultaneous fractional linear transformation

$$t \pm x \mapsto \frac{a + b(t \pm x)}{c + d(t \pm x)}$$

preserving the boundary

The conformal group of AdS is larger (Diff)

Far away from infinity means the following:

[here a picture showing causal region in the right half plane, projectinbg along its four edges yields four points a, b, c, d on the vertical axis that is the left boundary]

inavariant distance = conformal cross ratio

$$D = \frac{(a-d)(b-c)}{(a-b)(c-d)}$$

far away means $D \to \infty$

Conformal QFT on AdS_2 **= CFT on** M_+ Chiral fields

$$j^0(t,x) = j(t+x) + j(t-x)$$
$$j^1(t,x) = -j(t+x) + j(t-x)$$

$$D_\mu j^\mu = 0$$
$$j^1(t, x=0) = 0$$

Similar for other chiral fields, e.g. stress energy tensor (SET)

Field content of a one-dimensional CFT = one copy of current (SET) due to boundary condition at $x = 0$

Non-chiral fields Local fields on M_+

$$\Phi(t,x)$$

must commute with j^μ and with Φ at spacelike distance

Construction of such fields given below.

Behaviour at $D \to \infty$

Claim:

Far away from infinity, chiral fields behave like two fields

$$j_L(t+x), j_R(t-x)$$

non-chiral local fields behave like

$$\phi(t,x) = \sum a_l(t+x) \otimes a_R(t-x)$$

on the right non-local chiral blocks

This is the field content of two-dimensional CFT

A familiar model Neutral Weyl operators $\Phi(t,x) = e^{ij(f)}$

$$f(x) = G(x) - H(x)$$

$\Phi(t,x)$ commutes with $\Phi(t',x')$ whenever

$$t - x < t' - x' < t' + x' < t + x$$

the are therefore local fields on M_+

now look at asymptotic behaviour far away from $x = 0$ (the boundary)

[computation]

Correlations factorize (as $L \to \infty$) into products of "left" and "right" correlations of non-local chiral charged vertex operators

$$V_q(t+x) \otimes V_{-q}(t-x)$$

Lesson. Weyl operators

$$\Phi(t,x) = e^{ij(f)} = e^{iq} \int_{t-x}^{t+x} j(u)du$$

have interval localization from the 1D boundary perspective and a point localization from the 2D (M_+) perspective.

At this point there was some **discussion**

audience: the theory on the boundary is a bit weird, certainly not QM (i.e. 0+1 dim QFT) here, so what's the point if the lower dimensional theory is crazy?

Rehren: does make sense after all.

Basic observation: Assume a given 1-dimensional (crazy) chiral CFT. For $d < c < b < a$ let

$$O = \{(t,x) : t + x \in [b,a], t - x \in [d,c]\} \subset M_+$$

Consider all operators on the Hilbert space that are functions of the chiral fields smeared in the outer interbal $L = [d,a]$ and commute with all chiral fields in the inner interval $K = [c,b]$

Shorthand

$$I \vee J == I \times J$$

$$B_+(O) = A(K)' \cap A(L)$$

[I stopped taking notes at this point since this was entirely a review of the article with Longo: "Local fields in boundary conformal QFT"]

"algebraic holography is a higher dimensional version of the same general idea [of that article]"

Proposition: Bulk CFT = CFT on AdS from boundary CFT Every Moebius covariant system of local algebra $B_+(O)$ irreducibly containing a given subalgebra of chira fields is intermediate betwee

$$A(I) \vee A(J)$$

and

$$B(K)' \cap B(L)$$

where $A(I)$ are the local algebras (Rehren-Longo)

General theory For a given (completely rational) chiral local theory A, the relatively local (possibly non-local) irreducible "chiral extensions"

$$A(I) \subset B(I)$$

$I \subset \mathbb{R}$ can be classified. There are only finitely many

Each of themgoives rise to a maximal conformal theor yon M_+ defined by $B(K)' \cap B(L)$ om the Hilbert space B an drthe intermediate subtheoreis on the same Hilbert spoace can alsoe be classified THere are again only finitely many.

Split states. If O_1 and O_2 are two spacelike separated regions and ϕ_1 and ϕ_2 are two states on the local observables on O_1 and O_2, respectivel, one may ask:

is there a global state that extends the product state on the commuting algebras $A(O_1)$ and $A(O_2)$

$$\phi(\Phi_1 \Phi_2) = \phi_1(\Phi_1)\phi_2(\Phi_2)$$

?

The answer depends on the phase space properties (existence of partition sums, e.g.)

Split property: if a split states existsn, then there is a distinguished vector $\Xi \in H$ such that

$$(\Xi, A_1 A_2 \xi) = \phi_1(A_1)\phi_2(A_2)$$

moreover there is an algebra isomorphism

$$A(O_1) \vee A(O_2) \simeq A(O_1) \otimes A(O_2)$$

How to remove the boundary 1D vs 2D

The split property provides local isomorphism between

- $A(I) \vee A(J)$ (the chiral subalgebra of a CFT on M_+ generated by one set of chiral fields) and
- $A(I) \otimes A(J)$ (the chiral subalgebra of a CFT on M^2 generated by two sets of ..)

Proposition 1 (Modular Möbius covariance) *The modular groups of three algebras $A(I)$, $A(I_1)$, $A(I_2)$ $I_i \subset I$ in the vacuum state generate a unitary representation of the Moebius group.*

I is split in two subintervals
together with the split property

Proposition 2 *The modula groups of three algebras $A(I)$, $A(I_1)$, $A(I_2)$ and three algebra $A(J)$... in the split state generate a unitary representation of two Moebius groups.*

Recovering a 2D CFT on M^2 from the CFT on M_+

- Fix I and J, $O = I \times J \subset M_+$
- construct GNS representaiton of the (extended) split state of $B_+(O) \Rightarrow$ Hilbert space
$$H = \oplus_{ij} Z_{ij} H_i \otimes H_j$$
- construct unitary reps of Moeb times Moeb
- transport fields and algebras with this rep to al of M^2
- Check consistency (algebras not overdetermined?)
- check local commutativity

From AdS2 to M^2

$$AdS_2 \to M_+ \to (via boundary \mathbb{R} \to M^2)$$

Far away from ∞ far away from the boundary

- Fix I and J O as before
- Pick obsersables ϕ_i in $B_+(=)$ Shift "to the right" by $\alpha_L \in Moeb \times Moeb$
- Compute the vacuum expectation values in $L \to \infty$ limit
- by construction, Ξ is the GNS vacuum of the associated 2D CFT on M^2. Thus

Mechanism at work: clustering = decay law (here: chiral) correlation functions to the slit vector Ξ converges to the vacucum vector

From AdS2 to M^2 AdS2 to tiny regions to tangent space $= M^2$

conclusion

- CFT on AdS2 is CFT on halfspace M_+
- CFT on M_+ is made of the degrees of freedom of a chiral CFT on the boundary (bilocalized chiral fields by relative commutants)
- far away from the boiundary the degrees of freeodm of a full 2D CFT re-emerges thanks to split property and cluster behaviour
- the associated 2D CFT has different Hilbert space, different ground state and different Hamiltonian

Discussion audience: conformal invariance on AdS2 was assumed, conclusion in general?

Rehren: general d-dimensional case much more complicated

Rehren: algebraic holography predicts that going from AdS to boundary the boundary fields will violate the time-slice axiom (not mentioned here), one should expect anyway that CFT fields violate time-slice axiom. since CFT fields on boundary have continuous mass (?) so even without interaction the time-slice axiom is violated

audience: what is relation to usual AdS theory with gravity?

Rehren: cannot answer in satisfactory way, because not clear how algebras in net are generated from fields, so in this language no room for action

would be strange if nice relation established by algebraic holography is not related to usual AdS/CFT

audience: Lagrangian theories will satisfy time slice axiom

audience: gravity?

Rehren: on AdS background like ordinary field theory