## Graph Theory 2

Exercise Sheet 2

due on November 2, 1pm

http://bit.ly/2zPMMNz

**Exercise 1** ( $\S3.29$ ) Show that 2k-edge-connected graphs are k-edge-linked in the sense that for all distinct vertices  $s_1, \ldots, s_k, t_1, \ldots, t_k$  there are edge-disjoint paths  $P_i = s_i \ldots t_i$  for  $i = 1, \ldots, k$ . **Exercise 2** (§3.30) [1 Punkt] Show that k-linked graphs are (2k-1)-connected. Are they even 2k-connected? Are they 2k-connected if they have at least 2k + 1 vertices? **Exercise 3** (§3.31) [1 Punkt] For every  $k \in \mathbb{N}$  find an  $\ell = \ell(k)$ , as large as possible, such that not every  $\ell$ -connected graph is k-linked. **Exercise 4** (\$3.34)

Use Theorem 3.5.3 to show, that the function  $h: \mathbb{N} \to \mathbb{N}$  in Lemma 3.5.1 can be chosen as  $h(r) = cr^2$ , for some c.

## Written Exercise $(\S2.32)$

Show that if G is k-linked and  $s_1, \ldots, s_k, t_1, \ldots, t_k$  are not necessarily distinct vertices such that  $s_i \neq t_i$  for all *i*, then *G* contains independent paths  $P_i = s_i \dots t_i$  for  $i = 1, \dots, k$  i.e., the inner vertices of the paths  $P_1, \ldots, P_k$  are pairwise disjoint.

[1 Punkt]

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