Fachbereich Mathematik
Diskrete Mathematik
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WS 2012/13
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Probabilistic Method and Random Graphs

9. Series

due on January 22

Exercise 1
Show that for every $k \geq 2$ and $n^{-k/(k-1)} \ll p \ll n^{-(k+1)/k}$ a.a.s. the random graph $G(n,p)$ is a forest containing all trees on at most k vertices, but no larger tree.

Exercise 2
Show that if $p \ll n^{-1}$, then a.a.s. the random graph $G(n,p)$ is a forest.

Exercise 3
Show that for every $\varepsilon > 0$ the random graph $G(n,M)$ with $M \geq (1+\varepsilon)n$ is a.a.s. not planar.

Exercise 4
Show that for any integer $k \geq 1$ and any function $\omega(n)$ tending to infinity the following holds:

(a) If $pn < \ln n + (k-1)\ln\ln n - \omega(n)$, then a.a.s. $\delta(G(n,p)) \leq k-1$.

(b) If $pn > \ln n + (k-1)\ln\ln n + \omega(n)$, then a.a.s. $\delta(G(n,p)) \geq k$.