Fachbereich Mathematik
Diskrete Mathematik
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Probabilistic Method and Random Graphs

7. Series

due on Dezemeber 11

Exercise 1
Improve the probabilistic lower bound for the symmetric Ramsey number $R(n, n)$ from the first lecture.

Exercise 2
The *van der Waerden number* $W(r, k)$ is the least number n such that any coloring of $[n]$ with r colours yields a monochromatic arithmetic progression with k terms. Prove a lower bound on $W(r, k)$.

Exercise 3
Let $G = (V, E)$ be a cycle of length $4n$ and let $V = V_1 \dot\cup \ldots \dot\cup V_n$ be a partition of its vertex set into sets of size four. Is it true that there must be an independent set of G containing one vertex from each V_i? (Prove or supply a counterexample.)

Exercise 4
Show that a.a.s. $G(2n, p)$ contains a perfect matching if $p \gg n^{-1/3}$. Can show this for smaller p? (In fact, $p \gg \log n/n$ suffices.)