Fachbereich Mathematik
Diskrete Mathematik
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WS 2012/13
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Probabilistic Method and Random Graphs

6. Series

due on November 27

Exercise 1
Show that for any vectors $\boldsymbol{v}_1, \dots, \boldsymbol{v}_n \in \{-1, 1\}^n \subseteq \mathbb{R}^n$ there exist $\varepsilon_1, \dots, \varepsilon_n \in \{-1, 1\}$ such that the (Euclidean) norm of $\boldsymbol{v} = \sum_{i=1}^n \varepsilon_i \boldsymbol{v}_i$ is bounded by n.

Exercise 2
Improve the last exercise by showing that for any vectors $\boldsymbol{v}_1, \dots, \boldsymbol{v}_n \in \{-1, 1\}^n \subseteq \mathbb{R}^n$ and real numbers $\xi_1, \dots, \xi_n \in [0, 1]$ there exist $\varepsilon_1, \dots, \varepsilon_n \in \{0, 1\}$ such that for $\boldsymbol{u} = \sum_{i=1}^n \xi_i \boldsymbol{v}_i$ and $\boldsymbol{v} = \sum_{i=1}^n \varepsilon_i \boldsymbol{v}_i$ the (Euclidean) norm of $\boldsymbol{u} - \boldsymbol{v}$ is bounded by $n/2$. What is the connection to the last exercise?

Exercise 3
Show the following *one-sided Chebyshev inequality*, which asserts for any random variable X and $t > 0$ that

$$\mathbb{P}(X \geq \mathbb{E}X + t \cdot \sqrt{\mathbf{Var}\, X}) \leq \frac{1}{1 + t^2}\,.$$

Exercise 4
Show that for fixed $p \in (0, 1]$ the random graph $G(n, p)$ a.a.s. has diameter 2. Make a reasonable guess for the threshold for this property and try to prove it.