Fachbereich Mathematik
Diskrete Mathematik
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WS 2012/13
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Probabilistic Method and Random Graphs

5. Series

due on November 20

Exercise 1
Let X be a random variable. Show that:

(i) if X is nonnegative, then
$$\mathbb{P}(X=0) \leq \frac{\mathbf{Var}\, X}{\mathbb{E}[X^2]}$$

(ii) if $\mathbb{E}X = 0$, then for every $t > 0$ we have
$$\mathbb{P}(X \geq t) \leq \frac{\mathbf{Var}\, X}{\mathbf{Var}\, X + t^2}$$

Exercise 2
Show that every k-uniform hypergraph $H = (V, E)$ with $|E| \geq |V|/k$ contains an independent set $I \subseteq V$ of size
$$|I| \geq \left(1 - \frac{1}{k}\right)\left(\frac{|V|^k}{k|E|}\right)^{\frac{1}{k-1}}.$$

Exercise 3
Show that the *list chromatic number* of bipartite graphs with n vertices is at most $\lceil \log_2 n \rceil$, i.e., show that for any bipartite graph with vertex set V, $|V| = n$, and lists $(L_v)_{v \in V}$ with $L_v \subseteq \mathbb{N}$ and $|L_v| \geq \lceil \log_2 n \rceil$ there exists a coloring $f\colon V \to \mathbb{N}$ of the graph such that $f(v) \in L_v$ for every $v \in V$ and $f(u) \neq f(v)$ for every edge $\{u, v\}$.

Exercise 4
Show that every graph with m edges which contains a matching of size ν contains a bipartite subgraph with at least $(m + \nu)/2$ edges.

Hint: For Exercises 2-4 first moment method arguments suffice.