

Probabilistic Method and Random Graphs

4. Series

due on November 13

Exercise 1

Let $[n]_p$ denote the probability space of subsets of $[n]$, where each single element is included independently with probability p . What is the threshold for $[n]_p$ for the property of containing an arithmetic progression of length $k \geq 3$?

Exercise 2

Show that any monotone graph property has a threshold.

Exercise 3

For $1 > p > 0$ fixed and any graph F show that $G(n, p)$ contains *asymptotically almost surely* (a.a.s.), i.e., with probability tending to one as n tends to infinity, an induced copy of F . For which graphs F , containing an induced copy of F has a threshold?

Exercise 4

Let X be a random variable. Show that:

(i) if X is nonnegative, then

$$\mathbb{P}(X = 0) \leq \frac{\mathbf{Var} X}{\mathbb{E}[X^2]}$$

(ii) if $\mathbb{E}X = 0$, then for every $t > 0$ we have

$$\mathbb{P}(X \geq t) \leq \frac{\mathbf{Var} X}{\mathbf{Var} X + t^2}$$