Fachbereich Mathematik | WS 2012/13
Diskrete Mathematik | 23. Januar 2013
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Probabilistic Method and Random Graphs

10. Series

due on January 29

Exercise 1
Show that for every $\varepsilon > 0$ the random graph $G(n, M)$ with $M \geq (1 + \varepsilon)n$ is a.a.s. not planar. (*Hint:* Use Euler's formula adjusted for graphs of high girth.)

Exercise 2
Let $\omega(n)$ be function tending to infinity with n. Show that, if $2pn < \ln n + 2 \ln\ln n - \omega(n)$, then a.a.s. $G(n, p)$ contains at least two induced cherries.

Exercise 3
Let $\alpha > 0$ and suppose $0 < p = p(n) < 1$ is chosen in such a way that a.a.s. $G(n, p)$ contains a path of length αn.

(*i*) Show that there exists a $c > 0$ such that a.a.s. $G(n, p + c/n)$ contains a cycle of length $\alpha n/2$.

(*ii*) Can you replace "there exists $c > 0$" by "for every $c > 0$"? Can you strengthen "$\alpha n/2$" to "$(\alpha - o(1))n$"? Can you ensure both strengthening at the same time?

Exercise 4
Show that there is some $c > 0$ such that a.a.s. $G(n, c/n)$ contains no component with more than one cycle.