Fachbereich Mathematik
Diskrete Mathematik
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Probabilistic Method and Random Graphs

1. Series

due on October 23

Exercise 1
Show that every graph contains a bipartite subgraph with at least half of the edges. Give a probabilistic and a constructive argument.

Exercise 2
Give a constructive lower bound on the Ramsey number $R(n, n)$, i.e., give a "description" of a graph with many vertices which neither contains a clique of size n nor an independent set of size n. Can you find such a graph with more than n^2 vertices? Can you find a construction with number of vertices superpolynomial in n?

Exercise 3
Prove the following lower bound for the asymmetric Ramsey number

$$R(4, n) > c\left(\frac{n}{\log n}\right)^{3/2}$$

for some constant $c > 0$ and sufficiently large n.

Exercise 4
Suppose $A \in \mathbb{R}^{n\times n}$ has the property that all entries are distinct. Then there exists a permutation of the rows of A such that no column in the permuted matrix contains an increasing subsequence of length at least $c\sqrt{n}$ for some constant $c > 0$ and sufficiently large n.