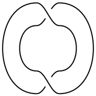
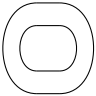
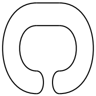
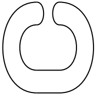
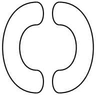
What is . . . Khovanov homology ?

Louis-Hadrien Robert

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

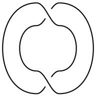
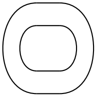
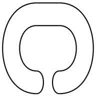
ZMP Seminar – DESY

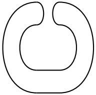
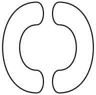
$\Big\langle$  $\Big\rangle = \Big\langle$  $\Big\rangle - q \Big\langle$  $\Big\rangle$

$- q \Big\langle$  $\Big\rangle + q^2 \Big\langle$  $\Big\rangle$

$$= (q + q^{-1})^2 - 2q(q + q^{-1}) + q^2(q + q^{-1})^2$$

$$= q^4 + q^2 + 1 + q^{-2}$$

$\Big\langle$  $\Big\rangle = \Big\langle$  $\Big\rangle - q \Big\langle$  $\Big\rangle$

$- q \Big\langle$  $\Big\rangle + q^2 \Big\langle$  $\Big\rangle$

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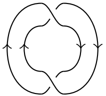
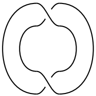
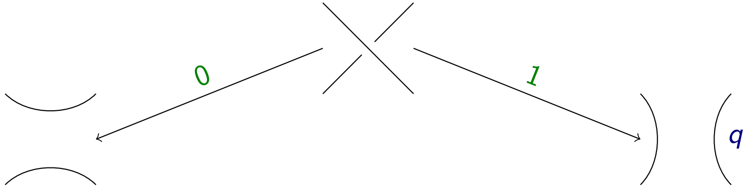
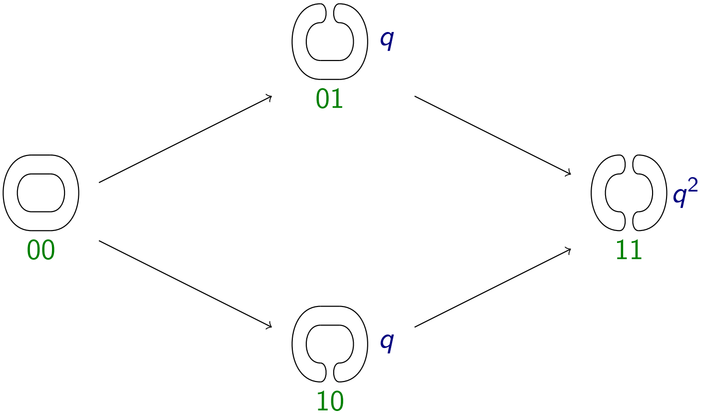
$J \Big($  $\Big) = q^2 \Big\langle$  $\Big\rangle \qquad (n_+, n_-) = (2, 0)$

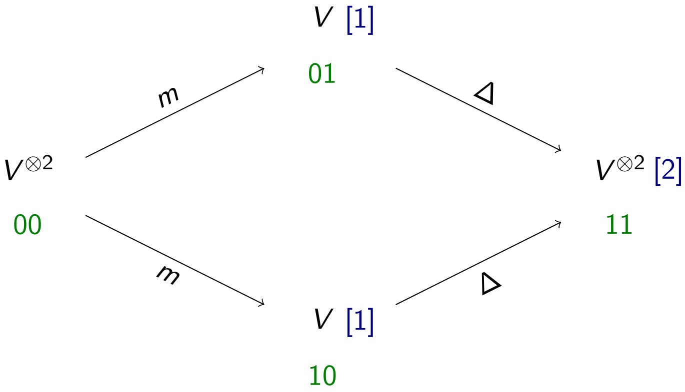
$$= q^6 + q^4 + q^2 + 1$$


0
1
q

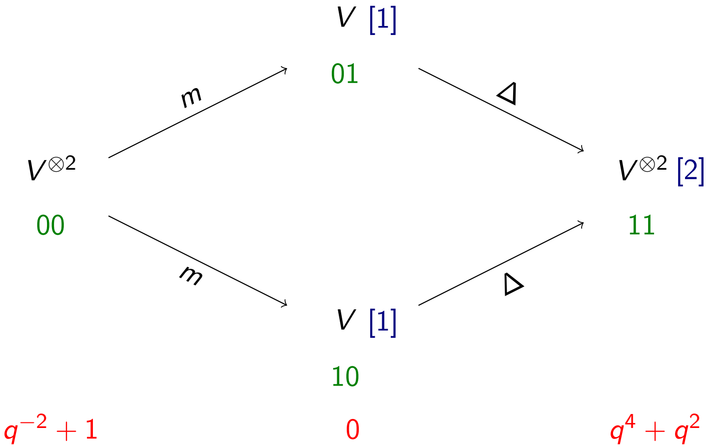
Khovanov homology



Khovanov homology



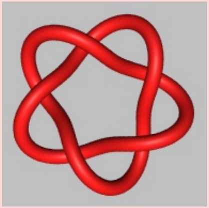
Khovanov homology



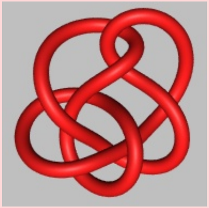
Shift the homological degree by $-n_-$, the q-degree by $n_+ - 2n_-$.
Take the homology: you juste computed the Khovanov homology.

Proposition (Bar-Natan, '02)

Khovanov homology is strictly stronger than the Jones polynomial.



5_1



10_{132}

(*source* www.colab.sfu.ca/KnotPlot/KnotServer/)

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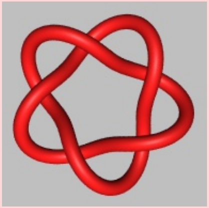

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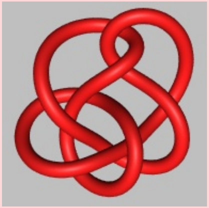

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Theorem (Kronheimer–Mrowka, '10)

Khovanov homology detects the unknot.

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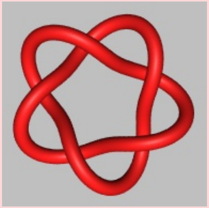

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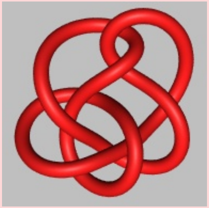

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Milnor conjecture (Kronheimer–Mrowka, '93, Rasmussen '04)

The slice genus of the (p, q)-torus knot is equal to $\frac{(p-1)(q-1)}{2}$.

Thank you!