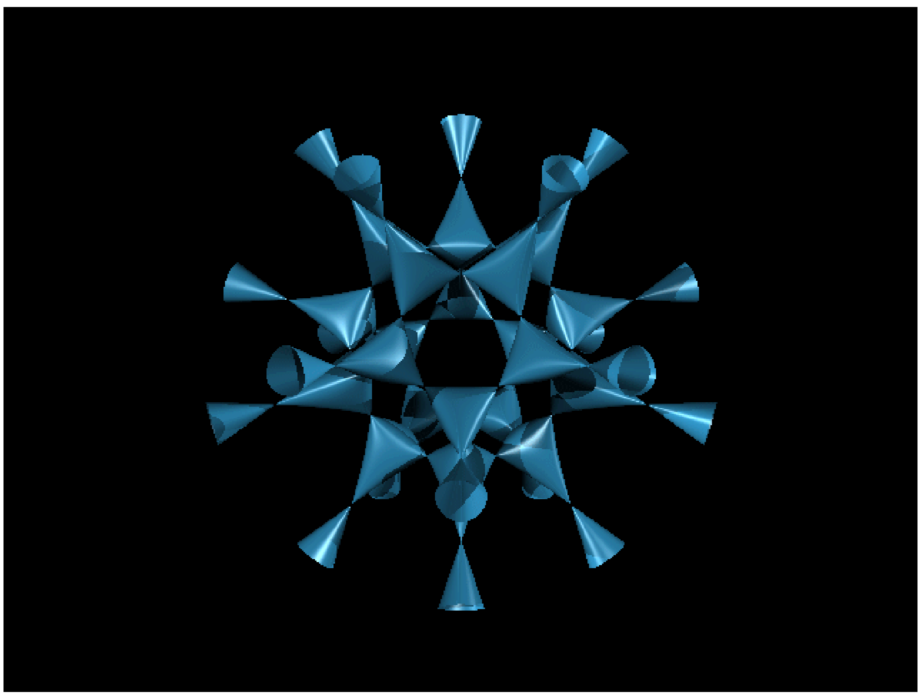
Singular Points of complex Analytic SurfaceS



Die Zeit ist / was ihr seyd
und ihr seyd / was die Zeit
Nur daß ihr wenger noch
als was die Zeit ist / seyd.

(Paul Fleming,
Gedanken über die Zeit)

After all, what can we ever gain in forever looking back and blaming ourselves if our lives have not turned out quite as we might have wished?

(Kazuo Ishiguro, *The Remains of the Day*)

Preface to the online Edition

If one adds the "large" capitals in the title

SINGULAR POINTS
OF COMPLEX
ANALYTIC SURFACES

I had chosen long time ago for this project one gets the German word „SPASS" (or in normal German writing without capitals: *„Spaß"*) which means "jest" or "joke", but also "joy". Planing the manuscript and writing the first chapters was real joy and fun, but later on it became something like a bad joke, even a nightmare.

I made several attempts to complete the manuscript and came close to publication around my 70th birthday in 2011, four years after my retirement. But then again the working process has been interrupted by further unexpected events in my life. Unfortunately, in the meantime all my *mathematical idols* which I mentioned in the 2011 Preface passed away: Professors Hans Grauert, Friedrich Hirzebruch, Reinhold Remmert and Egbert Brieskorn, even friends and colleagues of my age like Wolf Barth, and in Hamburg Peter Slodowy, Helmut Krämer and Johannes Michalicek.

My final decision now is to "publish" the manuscript on my *homesite* step by step mainly in the version of 2011, with some omissions compared to the Preface of 2011, but with some additional supplementary material in the main text as well as in the *Notes and References* attached to each chapter.

During the process the text will be quite "unstable". For instance, chapters may partially or completely be rewritten or grow substantially like, e.g., the *Supplement* that shall contain a comprehensive survey on the general theory of *sheaves* and its use in complex analytic geometry. As a consequence, the *Table of Contents* has gradually to be adapted as well as the cross-references have to be checked again. Moreover, I apologize for still missing or incorrect citations. In particular, English translations and comments on *literary citations*, an index of *music citations* and an index of *graphic art citations, photographs* etc. will be sampled at the end after completing the online manuscript.

I would like to invite everybody to criticize and to comment on the text and to make concrete proposals for improvements and additional material.

Hamburg, in March 2018 *Oswald Riemenschneider*

Singular Points of complex Analytic SurfaceS

An Introduction to the Local Analysis of Complex Analytic Spaces
(with special Emphasis to Dimension 2)

Oswald Riemenschneider
with the assistance of
Kurt Behnke

Hamburg 2011

Professor Dr. Oswald Riemenschneider
Fachbereich Mathematik der Universität Hamburg
Arbeitsgemeinschaft *Mathematisches Seminar*
Schwerpunkt *Analysis und Differentialgeometrie*
Bundesstraße 55
D 20146 Hamburg
e-mail riemenschneider@math.uni-hamburg.de

Demnach werde ich für alle Bücher, die ich geschrieben habe, zur Verantwortung gezogen?

(Hermann Hesse, *Der Steppenwolf*)

Preface

The first tentative plans and ideas for this book were developed more than three decades ago. More concrete conceptions arose in connection with a lecture that I planned to give during the summer semester 1977 but which due to other more urgent lecturing obligations did not take place. It was not until much later that my assistant Dr. Kurt Behnke was able to offer such a course in Hamburg in the 1980's (and later in Bonn in acting as a teaching substitute for Professor Egbert Brieskorn in 1991) with similar goals. His lecture notes have partly been worked into the final version of this book; they probably make up one quarter of the total content. The degree of his contribution not only to the realization of this project in particular but also to the management of all the tasks regarding research, teaching and organization associated with my research group during that time can hardly be underestimated. Finally, in winter semester 1996/97, I was able to give a four hours lecturing course in Hamburg about singularities; the (German) notes were partly translated and also included into this book.

Of course, complex analytic singularities have often been the subject of my seminar over the 25 years I have been active in Hamburg. In this way, colleagues, assistants and students over several generations have directly or at least indirectly accompanied and supported the evolution of this book. I am indebted to some of my colleagues in Hamburg for their contributions and encouragement, in particular to Professors Rolf Berndt, Helmut Brückner, the late Erich Kähler, Helmut Krämer, Johannes Michalicek, Helmut Müller, Peter Slodowy and Dr. Gerhard Mülich; from outside of Hamburg, I would like to mention Professors Michael Artin, David Eisenbud, Henry Laufer, Joseph Lipman, Henry Pinkham and Jonathan Wahl of the United States of America and Professors Shihoko Ishii, Hideaki Kazama, Kimio Miyajima, Iku Nakamura, Mutsuo Oka, Kyoji Saito, the late Nobue Sasakura, Ichiro Shimada, Tetsuo Suwa, Kei–ichi Watanabe, Kimio Watanabe and Tadashi Tomaru and Drs. Akira Ishii, Yukari Ito, Rie Kidoh and Miki Tsuji from Japan.

I also would like to thank my assistants - after Kurt Behnke I had the privilege and the pleasure to work with Dr. Jan Stevens, now in Gothenburg in Sweden, and Dr. Jörg Schürmann, now in Münster - and my students, some of them for their direct contributions in the form of seminar papers which in part have been worked into the text with or without substantial changes. Representatives of the group of such "involuntary" student collaborators to be named are Christian Adler, Jürgen Arndt, Jens Behrmann, Stephan Brohme, Heiko Cassens, Gunnar Dietz, Rüdiger Drewes, Gerhard Fender, Stefan Helmke, Jan–Erik Hoffmann, Michael Junge, Matthias Kabel, Constantin Kahn, Michael Koops, Andreas Leipelt, Stephan Mohrdieck, Jürgen Pesselhoy, Raimund Petow, Ancus Röhr, Harald Schnell, Wolfgang Scholz, Frank–Olaf Schreyer, Helmut Springstein, Christof Waltinger, Alf Werder, Michael Wöhrmann and Jürgen Wunram. Special thanks are going to Rüdiger Drewes and Andreas Leipelt who configurated in very efficient ways the PC's in my office and at home. Eckhardt Begemann advised me how to install and use MusiXTEX, and Martin Hamm helped me to prepare and insert some drawings.

If I am tracing back the roots of how this book came into being even further I feel strongly bound to thank three extraordinary mathematicians from whom I received enormously engraving impulses in the course of the semesters that I spent as a student and assistant at the University of Göttingen. First of all it's my pleasure to mention Professor Grauert to whose "Oberseminar" I was admitted just after taking my "Vordiplom" examination with him - I owe to him among many other more material forms of support the awakening of my enthusiasm for all aspects of complex analytic geometry that will serve me as background for singularity theory.

A stronger shaping of my algebraic inclinations was engendered and stimulated by Professor Remmert with whom I had the privilege to cooperate rather intensively during the elaboration of the first volume of the Grauert–Remmert trilogy. The most decisive impetus for the specific orientation of my later work, however, I obtained - rather belated in my own education - by Professor Brieskorns Göttingen lecture on *Simple Singularities* during summer semester 1973 that finally took me under the spell of *Special Singularities*. Some of the material which will be exposed on the pages of the present manuscript

I have seen in those days for the first time in a broader context. His unpublished Arcata lectures have been the promise to all experts, connoisseurs and dilettanti for the definitive text on singularity theory. The present manuscript doesn't make such heavy demands. Its single aim is to serve as an incomplete substitute for such an ultimate work and a partial complement to the existing literature.

From its genesis the text more or less urged me to keep its character of an introduction for students in middle and higher semesters. Therefore, the reader should be endowed with a good background in Algebra, Complex Analysis and Topology and moreover, if possible, with some basic knowledge of Complex Analytic and/or Algebraic Geometry. Already this list makes evident by what means "Singularity Theory" gains both its fascination for many experienced scholars and its reputation as being not approachable via the original literature for the "normal" student still in education: It lives a very distinct way of life among the most diversified mathematical branches - yet firmly rooted in and gathering strength from many of them which gives it the capability for fruitful interactions and applications. Or, to say it in the words of the authors of a recent text on *mirror symmetry*: we have *a wonderful story, but its telling requires lots of details in many different areas [...]*. I hope that I avoided some of the barriers on the steep path towards the reconnaissance of this complex terrain by following, at least partly, a genetic plan which leads from special examples to general theory and presents concepts and results from neighboring fields only when they are really needed.

With a view towards the exceedingly motivated and studious but rarely in all aspects of theoretical mathematics fully qualified student in Hamburg, I probably succumbed too much the temptation to utilize the text also as an introduction to the fields of, e. g, Several Complex Variables and Commutative Algebra by painstakingly formulating and explaining the vast variety of concepts from other fields which have to be taken into account. Such a concept will, without doubt, conjure up fatal boredom to the advanced reader who, therefore, will be well advised if he or she is skipping such paragraphs at first reading whose headings signal material he/she is familiar with or, perhaps even better, to glance over the text just in order to be informed about the terms and symbols used thereafter.

It is self–evident that this *principle of completeness* finds its limitations immediately in case one would try to insert without exception all the *proofs* of the results quoted and used in the manuscript. As a possible way out of this dilemma I have, at several places, chosen the method to state, without proof, the most spectacular general results and to deduce the useful corollaries from them, thus, in many cases, violating the correct historical and logical order. Those results which are not proven in the text are accentuated by an asterisk attached to the verb *Theorem* etc. As a rule, we will in such a case specify a bibliographic source in the *Notes and References* at the end of the Chapter. Here, we also include some information on the provenance of results and proofs in the main body of the text as well as hints to further reading. The Appendices attached to most of the Chapters - which may be omitted during first reading - serve mainly for documenting further results which complement and deepen the material under discussion.

We don't endeavor at this place to give a detailed survey of the content of the book. The initiated reader will gain sufficient information just by skimming through the extensive table of content whereas the novice would be confronted right now with a multitude of presumably incomprehensible words. May it suffice here to say that we place into the foreground *special classes* of *singular points of normal complex analytic surfaces* which are distinguished by especially nice constructions like *Klein singularities* and more generally *quotient surface singularities*, *cusp singularities*, *triangle singularities* etc. Their investigation, however, will in each case be embedded into the theory of larger classes like *rational* or *quasihomogeneous* or *minimally elliptic* singularities where as a main tool the cohomology theory of coherent analytic sheaves on resolutions of singularities will be brought into action.

From the last point of view LAUFER's monograph containing his 1969/70 Princeton lectures have much in common with my conception. In order to achieve a sufficiently strong compactness of my presentation it seemed absolutely necessary to integrate main subjects treated there into my considerations without rendering the reading of this charming booklet superfluous. With great profit I also consulted the books of BÄTTIG - KNÖRRER, DIMCA and OKUMA and occasionally they influenced my writing.

One important partial aspect of the program to *classify* all normal surface singularities - which is still in progress - consists in characterizing those (in a certain subclass, at least) which can be realized by only a single equation in three–dimensional affine space. In particular, the wondrously manifold manifestations of the *rational double points* require much space also in the present manuscript. They constitute a constantly recurrent "Leitmotiv" and its zenith but not the exclusive goal. Readers who wish a more direct and rapid access to this most carefully investigated class of singularities will study with great benefit the book of Lamotke who restricts himself to hypersurface singularities thus being able to dispense with too much technical effort. It contains, however, more on the relationship of the Klein singularities to the *Platonic solids* and Arnold's classification of the *simple* germs of functions.

By far the fastest but due its expected prerequisites very steep approach to the resolution of surface singularities is offered in the relevant passages of the "Ergebnisband" of Barth, Peters and Van De Ven. The reader who is more interested in *global* questions will find this an excellent address.

The readers who want to learn more on *catastrophe theory* in the sense of exploring germs of real valued *differentiable* maps may find the scope of our presentation disappointing. We strongly recommend to those the exquisite work of Arnold, Gusein–Zade und Varchenko. Much to my regret I had to leave out *deformation theory* from consideration almost completely since otherwise I had to expand the abstract apparatus immensely. Moreover, there exists at least in the case of isolated complete intersection singularities the outstanding text of Looijenga, and for more general classes which are placed in the foreground here this theory is still in a state of flux such that it seems impossible to present for the time being the ultimate version. Perhaps, a forthcoming book of de Jong and van Straten will cure this unsatisfactory state of affairs.

Thus, we are offering here a static picture of the singularities as if they were unchangeable precious stones which certainly does not full justice to their veritable character. Yet, even under this restricted view, they present a wealth of various facets which we hope to reproduce adequately or at least to illuminate as much that the reader will be enabled to discover their beauties himself. We further hope to supply the expert with a source of reference without aiming at full completeness and a text which may facilitate his burden to lead students up to the indispensable study of the original literature.

That, after all, this book didn't suffer the same fate as a great many projects which were begun with high enthusiasm but were never brought to an end is mainly due to one person and two institutions: I am deeply indebted to my secretary, Mrs. E. Dänhardt, for her never ceasing accuracy and skill in transforming again and again new drafts of the manuscript - in former times through the medium of the typewriter and later by use of the computer - into a readable and attractive shape and to my University for granting me regularly sabbatical semesters. I especially owe the *Stiftung Volkswagenwerk* a dept of gratitude for supplying me with extra sabbatical time by offering me an *Akademie–Stipendium*.

The reason for this book to having found its way only now into the scientific public lies in some private incidents and academic decisions which - to put it mildly - influenced the course of my life. This may account for one or the other resigned quotation as well as for the fact that these are borrowed from literary and musical works. As a rule I took the trouble to quote them in the original language even if I stumbled over them in a German translation. It shall be no wonder to the reader that citations in German predominate anyway.

Needless to say that I am well aware of the abundance of "Germanisms" in the mathematical text. If this Preface sounds more like "original" English it is the merit of my colleague William Kerby who translated my German draft.

I apologize to all the colleagues who waited much too long for a book which may not match their expectations in full measure and beg them for their understanding.

Hamburg, November 2011 *Oswald Riemenschneider*

How to use this book

The book is divided into chapters, each chapter into sections. The numbering of the sections will be continued in the appendices of the various chapters. Theorems, lemmas etc. will be numbered through each chapter. A reference "c.f. Section x" or "see Theorem y" refers to a section resp. a theorem of the chapter in which the reference is located whereas "c.f. Section a.b" refers to Section b in a different Chapter a. Definitions are not formally emphasized and thus, in the same way as with formulas, we do not affix a numeral to them. References to definitions and formulas will be accomplished by specifying the section in which they occur for the first time.

Each chapter will be concluded by some "Notes and References" which contain remarks on the history and provenance of results and on their discoverers and by calling the readers attention to relevant literature. In order to facilitate the finding of papers of a certain author we attach at the end of the book an alphabetic list of authors indicating the chapter(s) in which his/her works are documented. Here, as well as in the text, an entry [k.l] means item number l in the Notes and References supplementing Chapter k. Bibliographic details of the books which are mentioned in the preface to the book will be found at the end of the first chapter.

Prologue

Ich zum Beispiel habe, offen gestanden, meinen Schülern zeitlebens niemals ein Wort über den „Sinn" der Musik gesagt; wenn es einen gibt, so bedarf es meiner nicht. Dagegen habe ich stets großen Wert darauf gelegt, daß meine Schüler ihre Achtel und Sechzehntel hübsch genau zählten. Ob du nun Lehrer, Gelehrter oder Musikant wirst, habe die Ehrfurcht vor dem „Sinn", aber halte ihn nicht für lehrbar.

(Hermann Hesse, *Das Glasperlenspiel*)

Nur immer ein Buch zu schreiben, wenn man etwas Rundes zu sagen hat, ist menschlicher Stolz. Gibt es denn nicht noch mehr Figuren als die Runden, die auch alle schön sind? Die Schlangenlinie halte ich für ein Buch für die dienlichste. Und ich hatte schon in dieser Linie geschrieben, ehe ich wußte, daß Hogarth etwas über dieselbe geschrieben hatte, oder ehe Tristram Shandy seine Manier en Ziczac oder Ziczac à double Ziczac bekannt machte.

(Georg Christoph Lichtenberg)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Contents