

Previous Up

Citations From References: 0 From Reviews: 0

MR3643372 11M41 11Y60

Riemenschneider, Oswald (D-HAMB-SM)

Über einige elementare analytische Berechnungen von $\zeta(2)$. Variationen über ein Thema von Leonhard Euler. (German. German summary) [[Some basic analytical calculations of $\zeta(2)$. Variations on a theme by Leonhard Euler] *Mitt. Math. Ges. Hamburg* **36** (2016), 53–69.

The formula $\zeta(2) = \pi^2/6$ with $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ for s > 1 is important for some applications in analytic number theory, but its proofs often make use of Fourier analysis or complex analysis. This is a motivation to look for short elementary proofs, for example in D. Daners' paper [Math. Mag. 85 (2012), no. 5, 361–364; MR3007217].

Riemenschneider studies some basic analytical calculations of $\zeta(2)$, putting emphasis on historical sources of mathematics, namely from L. Euler [J. Litt. Allem. Suisse Nord 2 (1743), no. 1, 115–127; per bibl.; reprinted in P. Stäckel, Bibl. Math. (3) 8 (1907), 37-60; JFM 38.0061.03]. A method from [L. Euler, op. cit.] to calculate $\zeta(2)$ was also simplified in a letter from Euler to Daniel Bernoulli, and P. Levrie [Math. Intelligencer **33** (2011), no. 2, 29–32; MR2813260] made it into a valid proof. Euler also presented another calculation of $\zeta(2)$ in [op. cit.]. Riemenschneider is inspired by Levrie's paper [op. cit.], explains Euler's ideas and justifies convergence of corresponding series and integrals needed for the rigorous derivations of $\zeta(2)$. Matthias Kunik

© Copyright American Mathematical Society 2019