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Special representations and the two-dimensional McKay correspondence.

(English summary)

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The McKay correspondence for a finite subgroup G of $GL(2, \mathbb{C})$ is a one-to-one correspondence between the exceptional curves in the minimal resolution Y of \mathbb{C}^2/G and the nontrivial so-called (special) irreducible representations of G ; for G in $SL(2, \mathbb{C})$, every irreducible representation is special and one recovers the classical McKay correspondence.

In this article the author describes his previously unpublished work carried out in the late 1980s that characterises special representations of G . In addition, he uses the continued fraction construction of G -Hilb due to R. Kidoh [*Hokkaido Math. J.* **30** (2001), no. 1, 91–103; [MR1815001](#)] to establish an elementary proof of the McKay correspondence for a finite abelian subgroup G in $GL(2, \mathbb{C})$ (an independent proof via toric geometry was provided by Y. Ito [in *Geometry of toric varieties (Grenoble, 2000)*, 213–225, Soc. Math. France, Paris, 2002]). The article serves as a leisurely introduction to the excellent paper by A. Ishii [*J. Reine Angew. Math.* **549** (2002), 221–233; [MR1916656](#)] that established the McKay correspondence for finite G in $GL(2, \mathbb{C})$ as a fully faithful functor between bounded derived categories. *Alastair Craw*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.