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Kählersche Mannigfaltigkeiten mit hyper-q-konvexem Rand. (German)
Problems in analysis (Lectures Sympos. in honor of Salomon Bochner, Princeton Univ., Princeton, N.J., 1969), pp. 61–79. *Princeton Univ. Press*, *Princeton, N.J.*, 1970.

The vanishing theorems of K. Kodaira [Proc. Nat. Acad. Sci. U.S.A. **39** (1953), 1268–1273; MR0066693] and of Y. Akizuki and S. Nakano [Proc. Japan Acad. **30** (1954), 266–272; MR0066694] are generalized from compact Kähler manifolds to certain open Kähler manifolds in the following way. Let X be an n-dimensional complex manifold that is a relatively compact open set of the Kähler manifold $\hat{X}$. Suppose that ∂X, the boundary of X, has $\partial X=\{p=0\}$, with $dp\neq 0$ and p infinitely differentiable. X is called hyper-q-convex if, roughly speaking, for a suitable choice of local coordinates near each $x\in\partial X$, any q eigenvalues on ∂X at x of the complex Hessian of p have a positive sum. Hyper-q-convexity implies q-convexity and coincides with 1-convexity (strong pseudo-convexity) if $q=1$. Let V be a semi-negative, in the sense of T. Nakano [J. Math. Soc. Japan **7** (1955), 1–12; MR0073263], vector bundle over $\hat{X}$; if X is hyper-q-convex, then ${H_c}^s(X,\mathcal{V})=0$, $s\le n-q$, and $H^s(X,\mathcal{V}^*\otimes\Omega^n)=0$ for $s\ge q$. Here c denotes compact supports, $\mathcal{V}$ is the sheaf of germs of holomorphic sections of V, V^* is the dual of V, and Ω^n is the sheaf of germs of holomorphic n-forms.

The proof uses the direct sum decomposition of the space of C^∞ (r,s)-forms, $s\ge q$, due to J. J. Kohn [Ann. of Math. (2) **78** (1963), 112–148; MR0153030; ibid. (2) **79** (1964), 450–472; MR0208200]. The proof also yields more general results of a slightly technical nature. On compact manifolds, these results give Nakano's vanishing theorem [op. cit.] and the vanishing theorem of Akizuki and Nakano [op. cit.].

The authors give an example to show that q-convexity alone does not imply the vanishing theorem of this paper.

{For the entire collection, see MR0337450.} *H. B. Laufer*