Exercises in Algebraic Topology (master)

Prof. Dr. Birgit Richter

Summer term 2015

Exercise sheet no 6 For the exercise classes on the 13th and 20th of May 2015

21 (Standard examples) Write down explicit CW models for the 2-torus, for the Klein bottle and for the real and complex projective spaces.

22 (Rotten apple?) Is it possible to turn the Hawaiian earring into a CW complex? What about the cone on the Hawaiian earring?

23 (Attaching maps are important!)

(1) Are the homology groups of $\mathbb{S}^1 \times \mathbb{S}^1$ and of $\mathbb{S}^1 \vee \mathbb{S}^1 \vee \mathbb{S}^2$ isomorphic?
(2) What about the homology groups of their universal covering spaces?

24 (Euler characteristic) Let X be a finite CW complex. The *Euler characteristic* of X, $\chi(X)$, is then defined as

$$\chi(X) := \sum_{n \geqslant 0} (-1)^n \mathrm{rk}(H_n(X;\mathbb{Z})).$$

Here, rk denotes the rank of a finitely generated abelian group, *i.e.*, the number of its free summands.

a) What is $\chi(X)$ for a torus, a sphere or a general oriented compact closed surface of genus g, F_g?

b) What can you say about $\chi(X \sqcup Y)$ for two finite CW complexes X and Y? What about $\chi(X \cup Y)$ if X and Y are not necessarily disjoint?